概率论系列报告 Probability Seminar

报告题目(Title): Asymptotic Bounds on the Combinatorial Diameter of Random Polytopes

报告人(Speaker): Gilles Bonnet (University of Groningen)

时间(Time): **2022/05/23 14:00-15:00**

地点(Venue): Tencent meeting

摘要(Abstract): The combinatorial diameter $\operatorname{diam}(P)$ of a polytope P is the maximum shortest path distance between any pair of vertices. In this paper, we provide upper and lower bounds on the combinatorial diameter of a random "spherical" polytope, which is tight to within one factor of dimension when the number of inequalities is large compared to the dimension. More precisely, for an n-dimensional polytope P defined by the intersection of m i.i.d.\ half-spaces whose normals are chosen uniformly from the sphere, we show that $\operatorname{diam}(P)$ is $\Omega(n m^{\frac{1}{n-1}})$ and $O(n^2 m^{\frac{1}{n-1}} + n^5 4^n)$ with high probability when $m \geq 2^{\Omega(n)}$.
For the upper bound, we first prove that the number of vertices in any fixed two dimensional projection sharply concentrates around its expectation when m is large, where we rely on the $\Theta(n^2 m^{\frac{1}{n-1}})$ bound on the expectation due to

``shadows paths" together over a suitable net using worst-case diameter bounds to connect vertices to the nearest shadow. For the lower bound, we first reduce to lower bounding the diameter of the dual polytope P°, corresponding to a random convex hull, by showing the relation $\operatorname{diam}(P) \geq (n-1)(\operatorname{diam}(P^\circ)-2)$. We then prove that the shortest path between any ``nearly" antipodal pair vertices of P° has length $\Omega(m^{\frac{1}{n-1}})$.

This is a joint work with Daniel Dadush, Uri Grupel, Sophie Huiberts and Galyna Livshyts, see https://arxiv.org/abs/2112.13027.

欢迎参加

Everyone is welcome.