Quantile Regression

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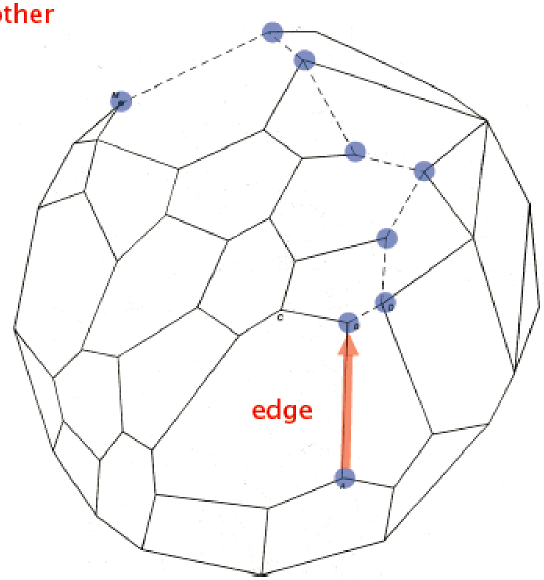
Simplex algorithm

Simplex algorithm. [George Dantzig 1947] Move from BFS to adjacent BFS, without decreasing objective function.

replace one basic variable with another



Greedy property. BFS optimal iff no adjacent BFS is better.
Challenge. Number of BFS can be exponential!

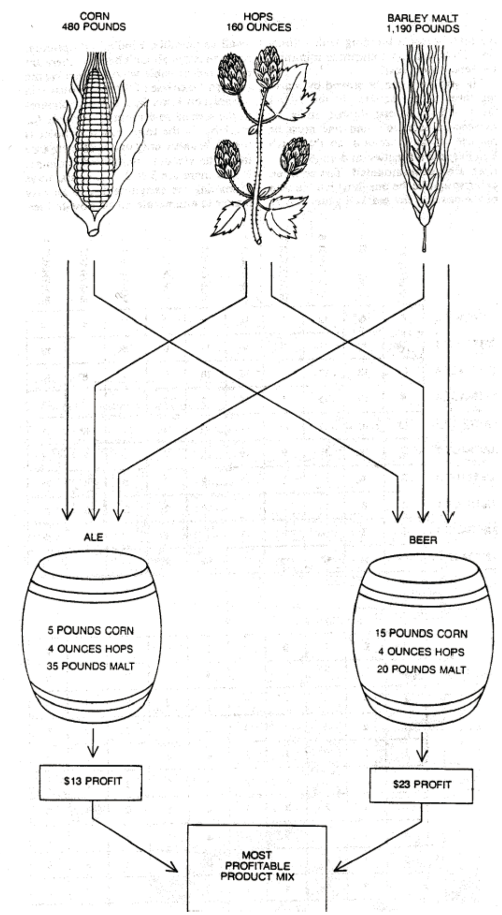
Brewery Example

objective function

	Ale		Beer			
max	$13A$	+	$23B$			Profit
s. t.	$5A$	+	$15B$	$\leq$	480	Corn
	$4A$	+	$4B$	$\leq$	160	Hops
	$35A$	+	$20B$	$\leq$	1190	Malt
	A	,	B	$\geq$	0	

constraint

decision variable



Simplex algorithm: Initialization

max Z subject to

$$\begin{array}{rcrcccccccrcr}
13A & + & 23B & & & & & & & - & Z & = & 0 \\
\hline
5A & + & 15B & + & S_C & & & & & & & = & 480 \\
4A & + & 4B & & & + & S_H & & & & & = & 160 \\
35A & + & 20B & & & & & + & S_M & & & = & 1190 \\
A & , & B & , & S_C & , & S_H & , & S_M & & & \geq & 0
\end{array}$$

Basis = $\{S_C, S_H, S_M\}$
$A = B = 0$
$Z = 0$
$S_C = 480$
$S_H = 160$
$S_M = 1190$

Simplex algorithm: Pivot (1)

max Z subject to

$$
\begin{array}{rcrcrcrcrcrcr}
13A & + & 23B & & & & & & & - & Z & = & 0 \\
\hline
5A & + & 15B & + & S_C & & & & & & & = & 480 \\
4A & + & 4B & & & + & S_H & & & & & = & 160 \\
35A & + & 20B & & & & & + & S_M & & & = & 1190 \\
A & , & B & , & S_C & , & S_H & , & S_M & & & \geq & 0
\end{array}
$$

Basis = $\{S_C, S_H, S_M\}$
$A = B = 0$
$Z = 0$
$S_C = 480$
$S_H = 160$
$S_M = 1190$

1. Why column 2?

Each unit increase in B increases objective value by $23

2. Why row 2?

Make sure the nonnegative constraints satisfied

Min{480/15,160/4,1190/20} = 480/15=32

Simplex algorithm: Pivot (2)

max Z subject to

$$\begin{array}{rcrcrcrcrcrcrcr}
13A & + & 23B & & & & & & & - & Z & = & 0 \\
\hline
5A & + & 15B & + & S_C & & & & & & & = & 480 \\
4A & + & 4B & & & + & S_H & & & & & = & 160 \\
35A & + & 20B & & & & & + & S_M & & & = & 1190 \\
A & , & B & , & S_C & , & S_H & , & S_M & & & \geq & 0
\end{array}$$

Basis $= \{S_C, S_H, S_M\}$
$A = B = 0$
$Z = 0$
$S_C = 480$
$S_H = 160$
$S_M = 1190$

Substitute: $B = 1/15\ (480 - 5A - S_C)$

max Z subject to

$$\begin{array}{rcrcrcrcrcrcr}
\frac{16}{3}A & & & - & \frac{23}{15}S_C & & & & & - & Z & = & -736 \\
\hline
\frac{1}{3}A & + & B & + & \frac{1}{15}S_C & & & & & & & = & 32 \\
\frac{8}{3}A & & & - & \frac{4}{15}S_C & + & S_H & & & & & = & 32 \\
\frac{85}{3}A & & & - & \frac{4}{3}S_C & & & + & S_M & & & = & 550 \\
A & , & B & , & S_C & , & S_H & , & S_M & & & \geq & 0
\end{array}$$

Basis $= \{B, S_H, S_M\}$
$A = S_C = 0$
$Z = 736$
$B = 32$
$S_H = 32$
$S_M = 550$

Simplex algorithm: Pivot (3)

max Z subject to

$$
\begin{array}{rcrcrcrcrcrcr}
\frac{16}{3}A & & & - & \frac{23}{15}S_C & & & & & - & Z & = & -736 \\
\hline
\frac{1}{3}A & + & B & + & \frac{1}{15}S_C & & & & & & & = & 32 \\
\frac{8}{3}A & & & - & \frac{4}{15}S_C & + & S_H & & & & & = & 32 \\
\frac{85}{3}A & & & - & \frac{4}{3}S_C & & & + & S_M & & & = & 550 \\
A & , & B & , & S_C & , & S_H & , & S_M & & & \geq & 0
\end{array}
$$

Basis = $\{B, S_H, S_M\}$
$A = S_C = 0$
$Z = 736$
$B = 32$
$S_H = 32$
$S_M = 550$

Substitute: $A = 3/8\ (32 + 4/15\ S_C - S_H)$

max Z subject to

$$
\begin{array}{rcrcrcrcrcrcr}
 & & & - & S_C & - & 2\ S_H & & & - & Z & = & -800 \\
\hline
 & & B & + & \frac{1}{10}S_C & + & \frac{1}{8}S_H & & & & & = & 28 \\
A & & & - & \frac{1}{10}S_C & + & \frac{3}{8}S_H & & & & & = & 12 \\
 & & & - & \frac{25}{6}S_C & - & \frac{85}{8}S_H & + & S_M & & & = & 110 \\
A & , & B & , & S_C & , & S_H & , & S_M & & & \geq & 0
\end{array}
$$

Basis = $\{A, B, S_M\}$
$S_C = S_H = 0$
$Z = 800$
$B = 28$
$A = 12$
$S_M = 110$