Proof that 1 = 0

Given any x, we have (by using the substitution $u = x^2/y$)

$$\int_0^1 \frac{x^3}{y^2} e^{-x^2/y}\, dy = \left[x e^{-x^2/y} \right]_0^1 = x e^{-x^2}.$$

Therefore, for all x,

$$\begin{aligned} e^{-x^2}(1 - 2x^2) &= \frac{d}{dx}(x e^{-x^2}) \\ &= \frac{d}{dx} \int_0^1 \frac{x^3}{y^2} e^{-x^2/y}\, dy \\ &= \int_0^1 \frac{\partial}{\partial x} \left(\frac{x^3}{y^2} e^{-x^2/y} \right) dy \\ &= \int_0^1 e^{-x^2/y} \left(\frac{3x^2}{y^2} - \frac{2x^4}{y^3} \right) dy. \end{aligned}$$

Now set $x = 0$; the left-hand side is $e^0(1 - 0) = 1$, but the right-hand side is $\int_0^1 0\, dy = 0$.

Ref.: Gelbaum and Olmstead, *Counterexamples in Analysis*, Dover, 2003