

MATH 251-019: Homework 9 (Due: 10/25/2017)

Please make your hand-writing clear to read. Please box your final answer.

1. Let $y(t)$ be the solution of the initial value problem

$$y'' + 3y' - 5y = u_\pi(t)e^{-6t}, \quad y(0) = -1, \quad y'(0) = 2.$$

Find its Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$.

- (A) $\frac{e^{-\pi s}}{(s+6)(s^2+3s-5)} - \frac{s+5}{s^2+3s-5}$
(B) $\frac{e^{-\pi(s+6)}}{(s+6)(s^2+3s-5)} - \frac{s+1}{s^2+3s-5}$
(C) $\frac{e^{-\pi(s+6)}}{(s+6)(s^2+3s-5)} - \frac{2s+5}{s^2+3s-5}$
(D) $\frac{e^{-\pi s}e^{6\pi}}{(s+6)(s^2+3s-5)} - \frac{s+1}{s^2+3s-5}$

2. Determine whether each statement below is **TRUE** or **FALSE**. You must justify your answers.

- (a) (Fall 2016, Exam 2, Question 7(b)) Suppose $f(t) = tu_2(t) - u_4(t)(t+1)$, then $f(3) = 3$.
(b) (Fall 2016, Exam 2, Question 7(c)) Suppose f is a continuous function such that $f(4) = 0$, then $\mathcal{L}\{\delta(t-4)e^{f(t)}\} = e^{-4s}$.
(c) (Spring 2016, Exam 2, Question 7(c)) $\mathcal{L}\{\delta(t-1)u_\pi(t)\cos(t)\} = \mathcal{L}\{\delta(t-3)u_\pi(t)e^{2t}\}$.

3. (Spring 2016, Exam 2, Question 10) Find the inverse Laplace transform of each function given below.

- (a) $F(s) = \frac{9-s}{s^2(s^2+9)}$
(b) $F(s) = e^{-9s} \frac{s-5}{s^2-4s+13} - \frac{\sqrt{7}}{2}$.

4. (Spring 2017, Exam 2, Question 10) Use the Laplace transform to solve the following initial value problem.

$$y'' - 3y' - 4y = u_2(t) - 2\delta(t-6), \quad y(0) = 0, \quad y'(0) = -2.$$

5. (Fall 2016, Exam 2, Question 9) Use the Laplace transform to solve the following initial value problem.

$$y'' - 2y' - 3y = 12u_6(t) + 4\delta(t - 2), \quad y(0) = 0, \quad y'(0) = 10.$$

6. (Spring 2016, Exam 2, Question 11) Use the Laplace transform to solve the following initial value problem.

$$y'' + 4y' + 3y = 2\delta(t - 1) + u_3(t)e^{2t-6}, \quad y(0) = 0, \quad y'(0) = 10.$$

7. (Fall 2015, Exam 2, Question 9) Suppose $\mathcal{L}\{f(t)\} = \frac{s}{s^4 + 10}$, and that $f(0) = 1, f(1) = 6, f(2) = -1, f(3) = -4$. Answer each question below.

- (a) Determine $\mathcal{L}\{tf(t)\}$.
- (b) Determine $\mathcal{L}\{f'(t)\}$.
- (c) Determine $\mathcal{L}\{e^{-3t}f(t)\}$.
- (d) Determine $\mathcal{L}\{\delta(t - 2)f(t)\}$.
- (e) Let $y(t) = \mathcal{L}^{-1}\{e^{-2s}\frac{s}{s^4 + 10}\}$. Evaluate $y(1)$.

8. Find the eigenvalues and eigenvectors of the following matrix.

(a) $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$