MATH 251-019: Homework 8 (Due: 10/18/2017)

Please make your hand-writing clear to read. Please box your final answer.

1. (Spring 2017, Exam 2, Question 2) Find the inverse Laplace transform of $F(s) = e^{-\pi s}\dfrac{2s+1}{s^2+2s+10}$.

   (A) $f(t) = u_\pi(t)(2e^{-t+\pi}\cos(3t-3\pi) - \dfrac{1}{3}e^{-t+\pi}\sin(3t-3\pi))$
   (B) $f(t) = u_\pi(t)(2e^{-t+\pi}\cos(3t-3\pi) - e^{-t+\pi}\sin(3t-3\pi))$
   (C) $f(t) = u_\pi(t)(2e^{t-\pi}\cos(3t-3\pi) - e^{t-\pi}\sin(3t-3\pi))$
   (D) $f(t) = u_\pi(t)(2e^{t-\pi}\cos(3t-3\pi) - \dfrac{1}{3}e^{-t+\pi}\sin(3t-3\pi))$

2. (Spring 2016, Exam 2, Question 3) Find the Laplace transform $\mathcal{L}\{u_\pi(t)e^{2t-2\pi}(t-\pi)^2\}$.

   (A) $\dfrac{2e^{-\pi s}}{(s-2)^3}$
   (B) $\dfrac{2e^{-2\pi}e^{-\pi s}}{(s-2)^3}$
   (C) $\dfrac{e^{-\pi s}}{s(s-2)^2}$
   (D) $\dfrac{e^{-\pi(2+s)}}{s(s-2)^2}$

3. (Spring 2017, Exam 2, Question 3) Find the Laplace transform $\mathcal{L}\{u_4(t)(t-2)^2\}$.

   (A) $F(s) = e^{-4s}\dfrac{2-12s+36s^2}{s^3}$
   (B) $F(s) = e^{-4s}\dfrac{2-4s+4s^2}{s^3}$
   (C) $F(s) = e^{-4s}\dfrac{2-4s+4s^2}{s^4}$
   (D) $F(s) = e^{-4s}\dfrac{2+4s+4s^2}{s^3}$

4. (Fall 2016, Exam 2, Question 2) Find the Laplace transform $\mathcal{L}\{u_1(t)e^{-3t}\sin(t-1)\}$

   (A) $F(s) = \dfrac{e^{3-s}}{(s-3)^2+1}$
   (B) $F(s) = \dfrac{e^{-3-s}}{(s-3)^2+1}$

(C) $F(s) = \dfrac{e^{-3-s}}{(s+3)^2+1}$

(D) $F(s) = \dfrac{e^{3-s}}{(s+3)^2+1}$

5. (Spring 2017, Exam 2, Question 4) Rewrite the following function using step functions:

$$f(t) = \begin{cases} 2t^2 - e^{-6t}, & t < 4 \\ 9t + 3, & 4 \le t < 10 \\ \sin(2t), & 10 \le t \end{cases}$$

(A) $f(t) = (2t^2 - e^{-6t})u_4(t) + (9t+3)(u_{10}(t) - u_4(t)) + \sin(2t)(1 - u_{10}(t))$
(B) $f(t) = (2t^2 - e^{-6t})(1 - u_4(t)) + (9t+3)(u_{10}(t) - u_4(t)) + \sin(2t)u_{10}(t)$
(C) $f(t) = (2t^2 - e^{-6t})(1 - u_4(t)) + (9t+3)(u_4(t) - u_{10}(t)) + \sin(2t)u_{10}(t)$
(D) $f(t) = (2t^2 - e^{-6t})u_4(t) + (9t+3)(u_4(t) - u_{10}(t)) + \sin(2t)(1 - u_{10}(t))$

6. (Fall 2016, Exam 2, Question 1) Consider the function

$$f(t) = \begin{cases} 0, & t < 3 \\ t^2, & 3 \le t < 5 \\ 3\cos t, & t \ge 5. \end{cases}$$

Which of the following expressions also describes $f(t)$?

(A) $f(t) = (u_5(t) - t_3(t))t^2 + u_5(t)3\cos t$
(B) $f(t) = u_3(t)t^2 + u_5(t)3\cos t$
(C) $f(t) = u_3(t)t^2 + u_5(t)(-t^2 + 3\cos t)$
(D) $f(t) = (t^2 - 3\cos t)u_3(t) + u_5(t)t^2$

7. (Fall 2016, Exam 2, Question 3) Let $y(t)$ be the solution of the initial value problem

$$y'' - 2y' + 3y = u_2(t) - u_4(t), \qquad y(0) = 0, y'(0) = 8.$$

Find its Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$.

(A) $Y(s) = \dfrac{e^{-2s}}{s(s^2-2s+3)} - \dfrac{s^{-4s}}{s(s^2-2s+3)} - \dfrac{8}{s^2-2s+3}$
(B) $Y(s) = \dfrac{e^{-2s}}{s^2-2s+3} - \dfrac{s^{-4s}}{s^2-2s+3} - \dfrac{8s}{s^2-2s+3}$
(C) $Y(s) = \dfrac{e^{-2s}}{s(s^2-2s+3)} - \dfrac{s^{-4s}}{s(s^2-2s+3)} + \dfrac{8}{s^2-2s+3}$
(D) $Y(s) = \dfrac{e^{-2s}}{s(s^2-2s+3)} - \dfrac{s^{-4s}}{s(s^2-2s+3)} + \dfrac{8s}{s^2-2s+3}$

8. Determine whether each statement below is **TRUE or FALSE**. You must justify your answers.

   (a) (Spring 2017, Exam 2, Question 7(c)) Suppose $f(t) = (t^2+4)u_e(t) - 7u_\pi(t)$, then $f(4) = 20$.

(b) (Spring 2016, Exam 2, Question 8(a)) Laplace transform has the following properties: $\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$, for any constants a and b; and that $\mathcal{L}\{f(t)g(t)\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$.

(c) (Spring 2016, Exam 2, Question (b)) The third derivatives of the Laplace transform $\mathcal{L}\{f(t)\}$ is given by $\dfrac{d^3}{ds^3}\mathcal{L}\{f(t)\} = -\mathcal{L}\{t^3 f(t)\}$.

(d) (Spring 2015, Exam 2, Question 8(b)) Suppose $f(t) = u_3(t)\cos t + u_2(t)t + u_4(t)$, then $f(\pi) = \pi$.

9. (Spring 2017, Exam 2, Question 9(a)) Find the inverse Laplace transform of $F(s) = e^{-2s}\dfrac{s^3+3}{s(s+1)^2}$.

10. (Fall 2016, Exam 2, Question 8(a)) Find the inverse Laplace transform of

$$F(s) = e^{-6s}\frac{5s+20}{s^3+2s^2+10s}.$$

11. (Fall 2014, Exam 2, Question 10) Find the Laplace transform of $f(s) = \begin{cases} t^2, & t < 4 \\ 2t + e^{\pi t}, & t \geq 4. \end{cases}$