MATH 251-019: Homework 7 (Due: 10/11/2017)

Please make your hand-writing clear to read. Please $\boxed{\text{box}}$ your final answer.

1. (Spring 2017, Exam 2, Question 1) Evaluate the following definite integral
$$\int_0^\infty t^3 e^{-(s-2)t} dt.$$
   (A) $\frac{6}{(s-2)^4}$
   (B) $\frac{3}{(s-2)^3}$
   (C) $\frac{3}{(s+2)^3}$
   (D) $\frac{6}{(s+2)^4}$

2. (Fall 2015, Exam 2, Question 1) Evaluate the following definite integral
$$\int_0^\infty e^{-(s+4)t} \cos(2t) dt.$$
   (A) $\frac{s+4}{(s+4)^2+4}$
   (B) $\frac{s}{(s+4)(s^2+4)}$
   (C) $\frac{s-4}{(s-4)^2+4}$
   (D) $e^{-4s}\frac{s}{s^2+4}$

3. (Spring 2015, Exam 2, Question 1) Evaluate the following definite integral
$$\int_0^\infty e^{-(s-3)t} \cos(2t) dt.$$
   (A) $\frac{(s-3)}{(s-3)^2+4}$
   (B) $\frac{s}{(s-3)(s^2+4)}$
   (C) $\frac{s}{(s+3)(s^2+4)}$
   (D) $\frac{s+3}{(s+3)^2+4}$

4. (Fall 2014, Exam 2, Question 1) Evaluate the following definite integral
$$\int_0^\infty te^{-st} \sin(4t) dt.$$
   (A) $\frac{8s}{(s^2+16)^2}$
   (B) $\frac{-s}{s^2+16}$
   (C) $\frac{2}{s^2(s^2+16)}$
   (D) $\frac{4}{(s^2+16)^2}$

5. (Spring 2014, Exam 2, Question 1) Evaluate the following definite integral

$$\int_0^\infty e^{-st}(t^3 + \sin(2t))dt.$$

(A) $\frac{3}{s^4} + \frac{2}{s^2+4}$
(B) $\frac{6}{s^4} + \frac{2}{s^2+4}$
(C) $\frac{6}{s^4} + \frac{1}{s^2+2^2}$
(D) $\frac{3}{s^3} + \frac{1}{s^2+2^2}$

6. For each part below, determine whether the statement is true or false. **You must justify your answers.**

   (a) (Spring 2017, Exam 2, Question 7(a)) Suppose $\mathcal{L}\{f(t)\} = \frac{s^4}{s^5+6}$, then $\mathcal{L}\{e^{-8t}f(t)\} = \frac{(s+8)^4}{(s+8)^5+6}$.

   (b) (Spring 2017, Exam 2, Question 7(b)) Suppose $\mathcal{L}\{f(t)\} = \frac{s^4}{s^5+6}$ and given $f(0) = 1$, then $\mathcal{L}\{f'(t)\} = \frac{-6}{s^5+6}$.

   (c) (Fall 2016, Exam 2, Question 7(a)) Suppose $\mathcal{L}\{\frac{s^2}{s^3+6}\}$, then $\mathcal{L}\{e^{2t}f(t)\} = \frac{(s+2)^2}{(s+2)^3+6}$.

   (d) (Fall 2015, Exam 2, Question 8(b)) $\mathcal{L}\{t(t-1)e^{2t-3}\} = e^{-3}\mathcal{L}\{t^2e^{2t}\} - e^{-3}\mathcal{L}\{te^{2t}\}$.

   (e) (Spring 2015, Exam 2, Question 8(a)) $\mathcal{L}\{e^{-(2+3t)}(\cos(t) - \sin(t))\} = \frac{1}{e^2}\mathcal{L}\{e^{-3t}\cos(t)\} - \frac{1}{e^2}\mathcal{L}\{e^{-3t}\sin(t)\}$.

   (f) (Fall 2014, Exam 2, Question 8(a)) Suppose $\mathcal{L}\{f(t)\} = \frac{s^2-1}{s^3+8}$, then $\mathcal{L}\{e^{3t}f(t)\} = \frac{s^2-6s+8}{(s-3)^3+8}$.

   (g) (Fall 2014, Exam 2, Question 8(b)) Let C_1 and C_2 be any 2 constants, $\mathcal{L}\{C_1f(t) - C_2g(t)\} = \mathcal{L}\{C_1\}\mathcal{L}\{f(t)\} - \mathcal{L}\{C_2\}\mathcal{L}\{g(t)\}$.

   (h) (Spring 2014, Exam 2, Question 7(b)) Suppose $\mathcal{L}\{f(t)\} = \frac{1}{1+s^2}$, then $\mathcal{L}\{tf(t)\} = \frac{2s}{(1+s^2)^2}$.

7. (Spring 2017, Exam 2, Question 9(b)) Find the Laplace transform

$$\mathcal{L}\{(3t + \pi^2)e^{2t}\sin(t)\}.$$

8. (Fall 2016, Exam 2, Question 8) Suppose that $\mathcal{L}\{f(t)\} = \frac{s^2}{s^3+6}$, and that $f(0) = -1, f'(0) = 2$.

   (a) Determine $\mathcal{L}\{tf(t)\}$.

   (b) Determine $\mathcal{L}\{f''(t)\}$.

9. (Spring 2015, Exam 2, Question 10) Suppose $\mathcal{L}\{f(t)\} = \frac{s}{s^3+11}$. What is $\mathcal{L}\{e^{-2t}tf(t)\}$?