MATH 251-019: Homework 6 (Due: 10/04/2017)

Please make your hand-writing clear to read. Please $\boxed{\text{box}}$ your final answer.

1. (Fall 2012, Exam 1, Question 13) Consider the second order nonhomogeneous linear equation
$$y'' - 2y' - 3y = e^{-t} + 3.$$
   (a) Find $y_c(t)$, the solution of its corresponding homogeneous equation.
   (b) Find its general solution.
   (c) What is the **form** of particular solution Y that you would use to solve the following equation using the Method of Undetermined Coefficients? **DO NOT ATTEMPT TO SOLVE THE COEFFICIENTS**.
$$y'' - 2y' - 3y = 7te^{-t}\sin 2t.$$

2. (Fall 2011, Exam 1, Question 14) Consider the second order nonhomogeneous linear equation
$$y'' - 4y' + 5y = e^{2t} - 10t.$$
   (a) Find its general solution.
   (b) What is the **form** of particular solution Y that you would use to solve the following equation using the Method of Undetermined Coefficients? **DO NOT ATTEMPT TO SOLVE THE COEFFICIENTS**.
$$y'' - 4y' - 5y = (t+1)e^{2t}\sin t + t^2.$$

3. (Spring 2015, Exam 1, Question 11) Consider the following list of differential equations:

$$\begin{aligned}
&A. && u'' + 4u' + 13u = 0\\
&B. && u'' - 4u' + 4u = 0\\
&C. && u'' + 6u' + 6u = \cos(t)\\
&D. && u'' + 5u' - 4u = 2\\
&E. && u'' + u = 0\\
&F. && u'' + 4u = \sqrt{3}\sin(4t)\\
&G. && u'' + 6u' + 9u = 0\\
&H. && u'' + 9u = 5\pi\cos(3t)
\end{aligned}$$

   Each of the equations above may or may not describe the displacement of a mass-spring system. Each question below has **exactly** one correct answer. The same equation may be reused to answer more than one question.

(a) Which equation describes a mass-spring system that is critically damped?

(b) Which equation describes a mass-spring system that is underdamped?

(c) Which equation describes a mass-spring system that is undergoing resonance?

(d) Which equation describes a mass-spring system that exhibits a simple harmonic motion?

(e) Which equation describes a mass-spring system whose motion crosses the equilibrium position at most once?

4. Consider the following list of differential equations:

$$
\begin{aligned}
&A. \quad u'' + 6u' + 13u = 0 \\
&B. \quad u'' - u = 2\cos(t) \\
&C. \quad u'' + 2u' + u = 0 \\
&D. \quad u'' + 4u = \pi\cos(\sqrt{2}t) \\
&E. \quad u' + 4u = 0 \\
&F. \quad u'' + 9u = (1+\sqrt{3})\sin(3t) \\
&G. \quad u'' + 15u = 0 \\
&H. \quad u'' - 4u' + 4u = 0
\end{aligned}
$$

Each of the equations above may or may not describe the displacement of a mass-spring system. Each question below has **exactly** one correct answer. The same equation may be reused to answer more than one question.

(a) Which equation describes a mass-spring system that is undergoing resonance?

(b) Which equation describes a mass-spring system that is critically damped?

(c) Which equation describes a mass-spring system that is underdamped?

(d) Which equation describes a mass-spring system that exhibits a simple harmonic motion?

(e) Which equation describes a mass-spring system whose motion crosses the equilibrium position at most once?

5. (Spring 2017, Exam 1, Question 6) Consider a certain mass-spring system described by the equation

$$3u'' + \gamma u' + ku = 0, \qquad \gamma \geq 0, \quad k > 0.$$

Answer the following questions. Be sure to justify your answer.

(a) Suppose $k = 12$. For what value(s) of γ would the system be critically damped?

(b) Suppose $\gamma = 6$ and $k = 9$. Will an solution of the equation cross the equilibrium position more than once?

(c) Suppose $\gamma = 12$ and $k = 15$. Find the quasi-period of the system.

(d) Suppose a force of $F(t) = 49\sin(\alpha t)$ is applied to the system ($\alpha > 0$), and given that $\gamma = 0$ and $k = 75$. What is the value(s) of α if the system exhibits resonance?

6. (Fall 2016, Exam 1, Question 9) Consider the fourth order linear equation

$$y^{(4)} + 6y'' + 9y = 0.$$

Which of the following is its general solution?

(A) $y = (C_1 + C_2 t)\cos\sqrt{3}t + (C_3 + C_4 t)\sin\sqrt{3}t$
(B) $y = (C_1 + C_2 t + C_3 t^2 + C_4 t^3)e^{-\sqrt{3}t}$
(C) $y = C_1\cos\sqrt{3}t + C_2\sin\sqrt{3}t + C_3 e^{\sqrt{3}t} + C_4 e^{-\sqrt{3}t}$
(D) $y = C_1 e^{-\sqrt{3}t} + C_2 t e^{-\sqrt{3}t} + C_3 e^{\sqrt{3}t} + C_4 t e^{\sqrt{3}t}$

7. (Fall 2015, Exam 1, Question 8) Find the general solution of the fourth order equation

$$y^{(4)} + 6y''' + 10y'' = 0$$

(A) $y = C_1 + C_2 t + C_3 e^{-3t}\cos t + C_4 e^{-3t}\sin t$
(B) $y = C_1 + C_2 t + C_3 e^{3t}\cos t + C_4 e^{3t}\sin t$
(C) $y = C_1 + C_2 t + C_3 e^{t}\cos 3t + C_4 e^{t}\sin 3t$
(D) $y = C_1 t + C_2 t^2 + C_3 e^{-3t}\cos t + C_4 e^{-3t}\sin t$

8. (Fall 2014, Exam 1, Question 10) Consider the fourth order linear equation

$$y^{(4)} - 10y'' + 25y = 0.$$

Which of the following is the general solution?

(A) $y = (C_1 + C_2 t)\cos\sqrt{5}t + (C_3 + C_4 t)\sin\sqrt{5}t$
(B) $y = C_1 e^{-\sqrt{5}t} + C_2 t e^{-\sqrt{5}t} + C_3 e^{\sqrt{5}t} + C_4 t e^{\sqrt{5}t}$
(C) $y = C_1 e^{-\sqrt{5}t} + C_2 t e^{-\sqrt{5}t} + C_3 t^2 e^{-\sqrt{5}t} + C_4 t^3 e^{-\sqrt{5}t}$
(D) $y = C_1 e^{-\sqrt{5}t} + C_2 e^{\sqrt{5}t} + C_3\cos\sqrt{5}t + C_4\sin\sqrt{5}t$