MATH 251-019: Homework 4 (Due: 09/20/2017)

Please make your hand-writing clear to read. Please $\boxed{\text{box}}$ your final answer.

1. Consider the autonomous differential equation

$$y' = \sin(y)(y+1)(y-3).$$

   (a) Find all of its equilibrium solutions.

   (b) Classify the stability of each equilibrium solution. Be sure to provide a clear reason for your answer.

   (c) Suppose $y(\pi) = \beta$ and $\lim_{t\to\infty} y(t) = \pi$, find all possible β.

   (d) Suppose $y(2) = -2$, find $\lim_{t\to\infty} y(t)$.

2. Consider the differential equation

$$-2x + 3x^2y^2 + (2x^3y + e^{-y})y' = 0.$$

   (a) Verify that it is an exact equation.

   (b) Find $f(x, y)$ such that

$$\frac{\partial f}{\partial x} = M(x, y), \qquad \frac{\partial f}{\partial y} = N(x, y).$$

   (c) Find the solution of this equation satisfying $y(1) = 0$. You may leave your answer in an implicit form.

3. (Spring 2016, Exam 1, Question 11) Consider the differential equation

$$(4x^3y^3 - ye^{xy} + x) + (3x^4y^2 - xe^{xy} - y)y' = 0.$$

   (a) Verify that it is an exact equation.

   (b) Find the solution of this equation satisfying $y(2) = 0$. You may leave your answer in an implicit form.

4. (Fall 2016, Exam 1, Question 7) Suppose $y_1(t)$ and $y_2(t)$ are two solutions of the second order linear equation

$$(2 + \sin(t))y'' - \cos(t)y' + y = 0.$$

   What is the general form of their Wronskian $W(y_1, y_2)$?

5. Determine whether each statement below is TRUE or FALSE.

(a) (Fall 2016, Exam 1, Question 10) The pair of functions $y_1(t) = e^t$ and $y_2(t) = 0$ can be a set of fundamental solutions of a certain second order linear equation $y'' + p(t)y' + q(t)y = 0$.

(b) (Spring 2017, Exam 1, Question 7) Suppose $y = C_1y_1(t) + C_2y_2(t)$ is a general solution of a certain second order linear equation $y'' + p(t)y' + q(t)y = 0$, then their Wronskian $W(y_1(t), y_2(t)) = 0$.

(c) (Spring 2017, Exam 1, Question 7) Suppose $y_1(t)$ is a solution of a certain second order nonhomogeneous linear equation $y'' + p(t)y' + q(t)y = \mathbf{g(t)}$, then $y_2(t) = Cy_1(t)$, where C is any constant, is always another solution of the same equation.

6. (Fall 2013, Exam 1, Question 4) Consider all nonzero solutions of the linear equation

$$y'' + 4y' - 5y = 0.$$

As $t \to \infty$, they will

(A) all approach 0.
(B) some approach ∞, all the rest approach $-\infty$
(C) some approach 0, some approach ∞, and some approach $-\infty$
(D) neither approach any limit, nor approach $\pm\infty$

7. (Fall 2016, Exam 1, Question 13) Consider the second order linear equation

$$y'' - 5y' - 6y = 0$$

(a) Find the general solution of the equation.

(b) Find the solution satisfying the initial conditions $y(455) = 3, y'(455) = -3$.

(c) For your answer from (b), determine $\lim_{t\to\infty} y(t)$.

8. (Fall 2015, Exam 1, Question 7) Let $y(t)$ be the solution of the initial value problem

$$y'' - y' - 20y = 0, \qquad y(0) = 2, \quad y'(0) = \beta.$$

Find the value of β for which $\lim_{t\to\infty} y(t) = 0$.

9. (Spring 2017, Exam 1, Question 11) Given that $y_1(t) = t^2 \ln t$ is a known solution of the linear differential equation

$$t^2y'' - 3ty' + 4y = 0, \qquad t > 0.$$

Use reduction of order to find the general solution of the equation.

10. (Spring 2015, Exam 1, Question 13) Given that $y_1(t) = t^2$ is a known solution of the linear differential equation

$$t^2y'' - 5ty' + 8y = 0, \qquad t > 0.$$

Use reduction of order to find the general solution of the equation.