MATH 251-019: Homework 3 (Due: 09/13/2017)

Please make your hand-writing clear to read. Please $\boxed{\text{box}}$ your final answer.

1. Existence and Uniqueness Theorem. Find the largest interval where the solution can be defined for the following problems.

   (a) $\sin(t)y' + \frac{\ln(t+1)}{t-1}y = e^t, \quad y(2) = 1$;

   (b) $(t^2 - 4)y' + (t - 1)y = \tan(t/2), \quad y(3) = 5$.

2. An object with mass $m = 1kg$ is thrown upward with initial velocity $v_0 = 5m/s$ from the roof of a building which is 20 meters high. Assume there is a force due to air resistance that is **proportional to** the velocity of the object with a positive constant of proportionality (coefficientcient of drag) $\gamma = 0.5$. You may use $g = 10m/s^2$ as the gravitational constant. Let **downward** be the positive direction.

   (a) Write an initial value problem (i.e. give an equation and an initial condition) that describes the velocity of this object. You do NOT need to solve the problem.

   (b) Solve the initial value problem to find $v(t)$.

   (c) Let $S(t)$ be the position of the ball at time t second. Find $S(t)$.

3. (Spring 2017, Exam 1, Question 10) Consider the autonomous differential equation
$$y' = -3y^2(y - 1)^3(y + 3).$$

   (a) Find all of its equilibrium solutions.

   (b) Classify the stability of each equilibrium solution. Be sure to provide a clear reason for your answer.

   (c) Suppose $y(\pi) = 2$, find $\lim_{t \to \infty} y(t)$.

   (d) Suppose $y(2017) = -3$, find $y(2016)$.

4. (Fall 2016, Exam 1, Question 12) Consider the autonomous differential equation
$$y' = y^2(y^2 - 25)(y + 1).$$

   (a) Find all of its equilibrium solutions.

   (b) Classify the stability of each equilibrium solution. Justify your answer.

   (c) If $y(3) = 2$, then what is $\lim_{t \to \infty} y(t)$?

(d) Suppose $y(37) = \beta$, and that $\lim_{t \to \infty} y(t) = 0$. Find all possible values of β.

5. (Spring 2013, Exam 1, Question 11) Consider the autonomous differential equation
$$y' = -y^4 + 16y^2.$$

   (a) Find all of its equilibrium solutions.

   (b) Classify the stability of each equilibrium solution. Justify your answer.

   (c) If $y(t)$ is a solution that satisfies $y(-1) = -4$, what is $y(0)$? Without solving the equation, briefly explain your conclusion.

   (d) If $y(t)$ is a solution that satisfies $y(3) = -3$, then what is $\lim_{t \to \infty} y(t)$?

6. (Summer 2016, Exam 1, Question 2) Consider the differential equation
$$y^4 - 3x^2 \tan(y) + (4xy^3 - x^3 \sec^2(y) + 2e^{2y})y' = 0.$$

   Verify that it is an exact equation.

7. (Spring 2017, Exam 1, Question 3) Find the values α and β for which the following equation is exact:
$$3x^2y + \alpha x^5 y^3 + (\beta x^3 + x^6 y^2)y' = 0.$$

8. (Fall 2016, Exam 1, Question 4) For what values of λ and ν will the equation
$$(\lambda x^3 + 12y^2 \cos x)y' + 9x^2 y + \nu y^3 \sin x = 0$$

   be and exact equation?