MATH 251-019: Homework 2 (Due: 09/06/2017)

Please make your hand-writing clear to read. Please $\boxed{\text{box}}$ your final answer.

1. In each problem, compute the integration.
   (a) $\int te^{3t}dt$
   (b) $\int te^{2t^2}dt$
   (c) $\int \frac{3}{(y-1)(y+2)}dy$
   (d) $\int \frac{1}{4-y^2}dy$
   (e) $\int \frac{1}{4+y^2}dy$
   (f) (optional) $\int e^{3t}\sin(4t)dt$

2. (Lecture Notes, Sec 12.2, Question 1) Find the general solution of the given differential equation. If an initial condition is given, find the particular solution which satisfies this initial condition.
   (a) $y' - 2y = e^{2t}, \quad y(0) = 4;$
   (b) $y' - 3y = 25\cos(4t);$
   (c) $y' = 2t(y - t^2);$
   (d) $y' - \frac{2}{t}y = \frac{t+1}{t}, \quad y(1) = -3.$

3. (Fall 2012, Exam 1, Question 9) Solve the initial value problem
$$y' = \frac{2 - 3x^2 + 8x^3}{2 + 2y}, \quad y(1) = -2.$$
   Give your answer in the **explicit** form. (Please use the change of variable $u = y + 1$ when calculating $\int 2 + 2ydy$)

4. Existence and Uniqueness Theorem. Find the largest interval where the solution can be defined for the following problems.
   (a) (Fall 2016, Exam 1, Question 2) $\cos(t)y' + \frac{t}{t+1}y = e^t, \quad y(-\pi) = 1;$
   (b) (Spring 2016, Exam 1, Question 2) $(t^2-\pi^2)y'+(t-1)y = \sin(2t), \ y(-2) = 5;$
   (c) (Fall 2015, Exam 1, Question 2) $(t^2 - 4)y' + t^2y = e^t, \quad y(-3) = 4;$
   (d) (Spring 2014, Exam 1, Question 4) $(2-t^2)y' + y = \ln(1+t), \quad y(0) = -1;$
   (e) (Fall 2012, Exam 1, Question 4) $(t^2 + t)y' + \frac{1}{t-4}y = e^{2t}, \quad y(2) = 3\pi;$

5. (Spring 2017, Exam 1, Question 8) An object of mass $4kg$ is moving along a straight line, propelled by a constant force of $7200N$. Suppose the object's drag coefficient is $2kg/m$, and that drag force is proportional to the **square** of the object's velocity. The object is initially moving at a velocity of $10m/s$.

   (a) Write an initial value problem (i.e. give an equation and an initial condition) that describes the velocity of this object. You do NOT need to solve the problem.

   (b) Is the equation in (a) a linear equation?

   (c) Is the equation in (a) a separable equation?

   (d) Approximately how fast will the object be moving after *a very long time*?

6. (Fall 2016, Exam 1, Question 5) A 900-gallon tank initially contains 200 gallons of water and 10 pounds of salt dissolved it. Water enters the tank at a rate of 5 gal/min with concentration 4 lb/gal in it. The well-mixed solution leaves at a rate of 1 gal/min. Write an initial value problem that models the change of the amount of salt $Q(t)$ inside the tank during the time interval $0 \leq t \leq 175$.

7. Suppose a 120-gallon tank initially contains 90 gal of water with 90 ounces of salt dissolved in it. Salt water (with a concentration of 2 oz/gal) enters the tank at a rate of 4 gal/min. The well-mixed solution flows out of the tank at a rate of 3 gal/min.

   (a) Set up an initial value problem that models the amount of dissolved salt in the tank at any time, **until the tank is full**.

   (b) Solve the initial value problem.

   (c) What is the concentration of the dissolved salt in the tank at any time before the tank is full?

   (d) How much salt is in the tank when it is full?