MATH 251-019: Homework 1 (Due: 08/30/2017)

1. In each part below, determine the order of the given differential equation; also state whether the equation is linear or nonlinear. If the equation is nonlinear, please point out the term that makes it nonlinear.

   (a) (Fall 2016, Exam 1, Question 1) $(y''')^2 + (y'')^4 + \sin(t)y = 0$

   (b) (Spring 2012, Exam 1, Question 1) $t^2 + t^3 y + \sin(t)y - yy' = 0$

   (c) (Spring 2011, Exam 1, Question 1) $y''' + ty' + (\cos^2 y)t = t^3$

   (d) $y' + t^2 = y$

   (e) $(1 + y^3)y'' + ty' + y = e^t$

   (f) (Fall 2011, Exam 1, Question 1) $y' - t^2 y^3 = 6t^2$

2. Verify that each given function is a solution of the differential equation:

   (a) $y(t) = -\frac{1}{t-1}$ for $y' = y^2$ and initial condition $y(0) = 1$

   (b) $y(t) = \frac{2}{1+e^{-rt}}$ for $y' = r(1 - \frac{y}{2})y$ and initial condition $y(0) = 1$

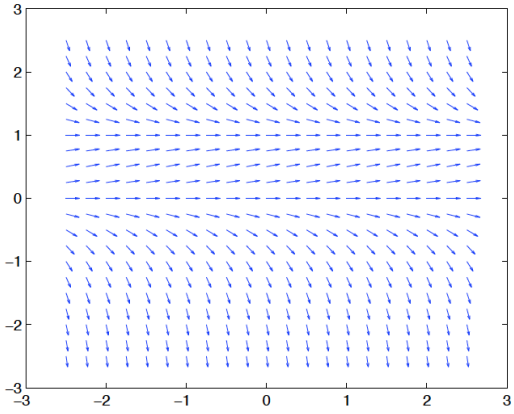
3. In each problem, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. Describe how this behavior depends on the initial value of y at $t = 0$.

   (a) $y' = y + 2$

   (b) $y' = y^2$

   (c) $y' = y(y - 2)^2$

4. (Fall 2015, Exam 1 Question 10a) Consider the following direction field of a certain first order equation:



(a) Which of the equations below does it represent? Why?
(A) $y' = y - x$
(B) $y' = y + x$
(C) $y' = y(1 - y)$
(D) $y' = x(1 - x)$

(b) Discuss the asymptotic behavior of y as $t \to \infty$.

5. Solve the following separable equations:

(a) (Spring 2017, Exam 1, Question 9) Solve the following initial value problem

$$(t^2 + 1)y' = \frac{1}{y - 4}, \qquad y(0) = 1.$$

Give your answer in the explicit form.

(b) (Fall 2016, Exam 1, Question 11) Solve the following initial value problem

$$y' = \frac{xe^{2x} + 3}{2y - 6}, \qquad y(0) = 2.$$

Give your answer in the explicit form.

6. (Spring 2013, Exam 1, Question 9) Consider the equation

$$2xy^3 + 3x^2y^2y' = 0.$$

Is it a separable equation? Find its general solution. You may leave your answer in implicit form (this means you do not have to find C).

7. Integrating factor. Please explain your solution step by step.

(a) (Fall 2016, Exam 1, Question 3) What is a suitable integrating factor for solving the following linear equation

$$(t^2 + 1)\frac{dy}{dt} + ty = te^{t^2}?$$

(A) $\mu(t) = (t^2 + 1)^{\frac{1}{2}}$
(B) $\mu(t) = \arctan t$
(C) $\mu(t) = e^{\frac{1}{3}t^3 + t}$
(D) $\mu(t) = e^t$

(b) (Spring 2014, Exam 1, Question 3) What is a suitable integrating factor that can be used to solve the equation

$$\cos(t)\frac{dy}{dt} + \sin(t)y = t^2, \qquad -\pi/t < t < \pi/2?$$

(A) $\mu(t) = \cos(t)$
(B) $\mu(t) = 1$
(C) $\mu(t) = e^{\cos(t)}$
(D) $\mu(t) = \frac{1}{\cos(t)}$