2024年秋，有限元方法II，上机作业1

截至时间：2024/12/8，晚上12点

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限18.
- 本次上机作业中, 须自己组装刚度矩阵, 推荐使用软件包iFEM. 请仔细阅读iFEM（或其他类似程序）中的实现方法，特别需要关注Matlab程序的向量化操作.
- 截止时间前将程序和上机报告的源码发送至snwu@math.pku.edu.cn

Consider the following second-order elliptic equation

$$\begin{cases} -\nabla \cdot (a(\boldsymbol{x})\nabla u) = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where the coefficient $a(\boldsymbol{x})$ satisfies the uniform ellipticity condition, i.e., there exist constants $\alpha_0, \alpha_1 > 0$ such that $\alpha_0 \le a(\boldsymbol{x}) \le \alpha_1$. In this lab, you are required to implement the $\mathcal{P}_3$ Hermite element.

Problem 1. On the uniform meshes over the domain $\Omega = [-1,1]^2$, consider a smooth coefficient $a(\boldsymbol{x}) = 1 + 0.5\sin(\pi x_1)\cos(\pi x_2)$. Choose a smooth solution u and compute f and g accordingly based on this smooth solution. Report the errors in H^1, L^2, W^1_∞, and L^∞ norms to verify the correctness of your code.

Problem 2. On a uniform mesh over the L-shaped domain $[-1,1]^2 \setminus [0,1]\times[-1,0]$, choose $a(x) = 1$ and the exact solution

$$u = (1-r^2)v(r,\theta), \quad v(r,\theta) = r^{\frac{2}{3}}\sin\left(\frac{2}{3}\theta\right). \tag{2}$$

Report the errors in H^1, L^2, W^1_∞, and L^∞ norms.

Problem 3 On a uniform mesh over the L-shaped domain, consider a given right-hand side $f = 1$ and boundary condition $g = 0$ (in this case, the exact solution is unknown). Freely choose some smooth functions $a(x)$ that meet the uniform ellipticity condition, and evaluate the accuracy of your code in different norms. (In this case, a reference solution can be obtained on a very fine mesh.)

Problem 4. Use adaptive meshes (please describe your algorithms for ESTIMATE, MARK, and REFINE): Report the convergence histories for Problem 2 & 3.