

2024秋，有限元方法II，作业2

交作业时间：2024/10/24

The Mathematical Theory of Finite Element Methods:

- Chapter 3: 3.x.10, 3.x.12, 3.x.13, 3.x.15, 3.x.18, 3.x.35
- Chapter 4: 4.x.6, 4.x.12, 4.x.16

Supplementary Questions:

1. Let $(K, \mathcal{P}, \mathcal{N})$ be a finite element satisfying
 - (a) There exists $\sigma > 0$ such that $\sigma h_K \leq \rho_K \leq h_K$;
 - (b) $\mathcal{P}_{m-1} \subset \mathcal{P} \leq W_\infty^m(K)$;
 - (c) $\mathcal{N} \subset (C^l(\bar{K}))'$;
 - (d) $(K, \mathcal{P}, \mathcal{N})$ is affine-interpolation equivalent to the reference finite element $(\hat{K}, \hat{\mathcal{P}}, \hat{\mathcal{N}})$.

Here, $m + l - n/p > 0$. For $0 \leq j \leq m - 1$, the constant t_j satisfies

$$\begin{cases} p \leq t_j \leq \frac{(n-1)p}{n-(m-j)p} & \text{if } (m-j)p < n, \\ p \leq t_j < \infty & \text{if } (m-j)p = n, \\ p \leq t_j \leq \infty & \text{if } (m-j)p > n. \end{cases}$$

Then, there exists a constant C independent of K such that, for any $0 \leq j \leq m - 1$ and $v \in W_p^m(K)$

$$\sum_{|\alpha|=j} \|\partial^\alpha(v - Iv)\|_{L^{t_j}(\partial K)} \leq C h_K^{m-j+(n-1)/t_j-n/p} |v|_{W_p^m(K)}.$$