2023年秋，有限元方法II，上机作业1

截至时间：2023/12/12，晚上12点

要求:

- 用TeX写上机报告(中英文均可)，包含必要的数值结果讨论，页数上限15.
- 本次上机作业中，须自己组装刚度矩阵，推荐使用软件包iFEM. 请仔细阅读iFEM（或其他类似程序）中的实现方法，特别需要关注Matlab程序的向量化操作.
- 截止时间前将程序和上机报告的源码发送至snwu@math.pku.edu.cn

Consider the following biharmonic equation

$$\begin{cases} (-\Delta)^2 u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g_0 & \text{on } \partial\Omega, \\ \dfrac{\partial u}{\partial \boldsymbol{n}} = g_1 & \text{on } \partial\Omega. \end{cases} \tag{1}$$

This lab entails the implementation of Morley elements on a triangular mesh and the solution of the above equation on a quasi-uniform grid. During numerical tests, the source function f and boundary data g_i can be derived from the exact solution u.

1. For the domain $\Omega = (0,1)^2$, select a suitable smooth solution u, compute the errors in discrete H^2 semi-norm, discrete H^1 semi-norm, and L^2 norm, then summarize the convergence orders.

2. For the L-shape domain $\Omega = (-1,1)^2 \setminus [0,1) \times (-1,0]$, the solution is set to be
$$u = r^{1.5}\sin(1.5\theta),$$
where (r,θ) are polar coordinates. Compute the errors in discrete H^2 semi-norm, discrete H^1 semi-norm, and L^2 norm, then summarize the convergence orders.

3. (optional) Consider the following H^2 perturbation problem:

$$\begin{cases} \varepsilon^2 \Delta^2 u - \Delta u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g_0 & \text{on } \partial\Omega, \\ \dfrac{\partial u}{\partial \boldsymbol{n}} = g_1 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

where $0 < \varepsilon < 1$. Evaluate the computational performance of Morley elements for this problem across different values of ε. In particular, what observations can be made from numerical tests when $\varepsilon \ll 1$?