

# 2023秋, 有限元方法II, 作业3

交作业时间: 2023/11/16

The Mathematical Theory of Finite Element Methods:

- Chapter 4: 4.x.10, 4.x.12
- Chapter 5: 5.x.8, 5.x.9, 5.x.16, 5.x.17, 5.x.21
- Chapter 9: 9.x.19

Supplementary Questions:

1. Conduct the duality argument for the IPDG scheme

$$a_h^-(u_h, v_h) = (f, v_h) \quad \forall v_h \in V_h,$$

where  $V_h = \{v \in L^2(\Omega) : v|_T \in \mathcal{P}_1(T) \ \forall T \in \mathcal{T}_h\}$ .

2. Given a triangle  $T$  with diameter  $h_T$ , if  $r \in \mathbb{R}$ , show that

$$h_T \|r\|_{L^2(T)} \lesssim \|r\|_{H^{-1}(T)},$$

where the hidden constant depends on the shape-regularity of  $T$ .