2023秋，有限元方法II，作业1

交作业时间：2023/09/28

The Mathematical Theory of Finite Element Methods:

- Chapter 0: 0.x.6, 0.x.10, 0.x.11, 0.x.12, 0.x.13
- Chapter 1: 1.x.13, 1.x.25, 1.x.27, 1.x.42

Supplementary Questions:

1. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain. There exists a constant $C(n)$ depending only on n such that for any $0 \leq \lambda < n$,

$$\max_{x\in\Omega} \int_\Omega |x-y|^{-\lambda} \mathrm{d}y \leq C(n)(n-\lambda)^{-1}|\Omega|^{1-\lambda/n}.$$

2. Let $\Omega \subset \mathbb{R}^n$ be a bounded Lipschitz domain. Show that for any $q \geq 1$,

$$\|v\|_{L^q(\Omega)} \leq C(n) q^{1-\frac{1}{n}} |\Omega|^{\frac{1}{q}} \|v\|_{W^{1,n}(\Omega)} \qquad \forall v \in W_0^{1,n}(\Omega).$$