2022年秋，有限元方法II，上机作业1

截至时间：2022/11/25，晚上12点

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限15.
- 本次上机作业中, 须自己组装刚度矩阵, 推荐使用软件包iFEM.
- 截止时间前将程序和上机报告的源码发送至snwu@math.pku.edu.cn

Consider the following Poisson equation

$$\begin{cases} -\Delta u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g & \text{on } \partial\Omega. \end{cases} \tag{1}$$

The computational domain is given as,

$$\Omega := \{(x, y) \in (-1, 1)^2 : 0 < \theta < \pi/\beta\},$$

where $\beta \geq \frac{1}{2}$. Note that if $\beta < 1$, then Ω is not convex. Use $\mathcal{P}_1$ and $\mathcal{P}_2$ Lagrange elements to solve (1) with different β's and exact solutions on both *quasi-uniform* and *adaptive* meshes. The source f and boundary data g can be obtained from the exact solution u.

Problem 1. Quasi-uniform meshes: Choose *smooth* solution u. Report the errors in H^1, L^2, W^1_∞ and L^∞ norms for different β's.

Problem 2. Quasi-uniform meshes: Choose

$$u = (1 - r^2)v(r, \theta), \quad v(r, \theta) = r^\beta \sin(\beta\theta). \tag{2}$$

Report the errors in H^1, L^2, W^1_∞ and L^∞ norms for different β's.

Problem 3. Adaptive meshes (please describe your algorithms for ESTIMATE, MARK, and REFINE): Report the convergence histories for singular solution (2).