2022秋，有限元方法II，作业1

交作业时间：2022/09/23

The Mathematical Theory of Finite Element Methods:

- Chapter 0: 0.x.6, 0.x.10, 0.x.11, 0.x.12, 0.x.13
- Chapter 1: 1.x.5, 1.x.13, 1.x.25, 1.x.42

Supplementary Questions:

1. For any $0 < s < 1$, show that

$$\int_{\mathbb{R}^n} \frac{|\mathrm{e}^{i\xi\cdot\eta} - 1|^2}{|\eta|^{n+2s}} \mathrm{d}\eta = \frac{1}{c(n,s)} |\xi|^{2s}.$$

   where the positive constant is given by

$$c(n,s) = \frac{2^{2s} s \Gamma(s + n/2)}{2\pi^{n/2} \Gamma(1-s)}.$$

2. Let $\Omega \subset \mathbb{R}^n$ be a bounded Lipschitz domain. Show that

$$\|v\|_{L^2(\partial\Omega)} \leq C(\Omega) \left(\epsilon^{-1} \|v\|_{L^2(\Omega)} + \epsilon \|v\|_{H^1(\Omega)}\right),$$

   for any $v \in H^1(\Omega)$ and $\epsilon \in (0,1)$.

3. Let $\Omega \subset \mathbb{R}^n$ be a bounded Lipschitz domain. Show that for any $q \geq 1$,

$$\|v\|_{L^q(\Omega)} \leq C(n) q^{1-\frac{1}{n}} |\Omega|^{\frac{1}{q}} \|v\|_{W^{1,n}(\Omega)} \qquad \forall v \in W_0^{1,n}(\Omega).$$