2021年秋，有限元方法II，上机作业2

截止日期：2022/01/16

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限15.
- 本次上机作业包含三种方法, 选一种实现即可.
- 本次上机作业不限制程序语言与软件包. 若选第三种方法并且刚度矩阵非自己组装, 请推导出该方法对应的误差指示子并数值验证其有效性与可靠性.
- 截止时间前将程序和上机报告的源码发送至 snwu@math.pku.edu.cn

Consider the Stokes equation

$$\begin{cases} -\mu\Delta \underset{\sim}{u} + \nabla p = \underset{\sim}{f} & \text{in } \Omega \subset \mathbb{R}^2, \\ -\nabla \cdot \underset{\sim}{u} = 0 & \text{in } \Omega, \\ \underset{\sim}{u} = \underset{\sim}{g}_D & \text{on } \Gamma_D, \\ 2\mu\boldsymbol{\varepsilon}(\underset{\sim}{u})\underset{\sim}{n} - p\underset{\sim}{n} = \underset{\sim}{g}_N & \text{on } \Gamma_N = \partial\Omega \setminus \Gamma_D. \end{cases} \tag{1}$$

Implement one of the following options. Then,

1. Test the velocity errors in H^1, L^2, and pressure errors in L^2 through appropriate data.
2. Simulate the 2D lid-driven cavity: $\Omega = (0,1)^2$, $\underset{\sim}{f} = \underset{\sim}{0}$, the fluid movement in the cavity is induced by the imposed boundary condition $\underset{\sim}{u} = (1,0)^T$ on the boundary $y = 1$ (and zero velocity on the other boundaries).
3. Choose $\mu = 1, 10^{-2}, 10^{-4}, 10^{-6}$, report the dependence of μ for the H^1 velocity errors $\|\nabla(\underset{\sim}{u} - \underset{\sim}{u}_h)\|_{L^2}$. What is the dependence of μ for other errors?

Option 1. Discrete Stokes complex; See [1].

Option 2. Rational bubble function; See [2].

Option 3. $H(\mathrm{div})$ + penality; See [3].

References

[1] Ricard S. Falk, and Michael Neilan. *Stokes complexes and the construction of stable finite elements with pointwise mass conservation*, **SIAM Journal on Numerical Analysis**, 51(2), 1308-1326, 2013.

[2] Johnny Guzmán, and Michael Neilan. *Conforming and divergence-free Stokes elements on general triangular meshes*, **Mathematics of Computation**, 83(285), 15-36, 2014.

[3] Blanca Ayuso de Dios, Franco Brezzi, L. Donatella Marini, Jinchao Xu, Ludmil Zikatanov. A simple preconditioner for a discontinuous Galerkin method for the Stokes problem, **Journal of Scientific Computing**, 58, 517-547, 2014.