2021秋，有限元方法II，作业2

交作业时间：2021/11/12

The Mathematical Theory of Finite Element Methods:

- Chapter 3: 3.x.15, 3.x.18, 3.x.35
- Chapter 4: 4.x.10, 4.x.12
- Chapter 5: 5.x.8, 5.x.9, 5.x.17, 5.x.21

Supplementary Questions:

1. Let $(K, \mathcal{P}, \mathcal{N})$ be a finite element satisfying
   (a) There exists $\sigma > 0$ such that $\sigma h_K \le \rho_K \le h_K$;
   (b) $\mathcal{P}_{m-1} \subset \mathcal{P} \le W^m_\infty(K)$;
   (c) $\mathcal{N} \subset (C^l(\bar{K}))'$;
   (d) $(K, \mathcal{P}, \mathcal{N})$ is affine-interpolation equivalent to the reference finite element $(\widehat{K}, \widehat{\mathcal{P}}, \widehat{\mathcal{N}})$.

   Here, $m + l - n/p > 0$. For $0 \le j \le m-1$, the constant t_j satisfies

   $$\begin{cases} p \le t_j \le \frac{(n-1)p}{n-(m-j)p} & \text{if } (m-j)p < n, \\ p \le t_j < \infty & \text{if } (m-j)p = n, \\ p \le t_j \le \infty & \text{if } (m-j)p > n. \end{cases}$$

   Then, there exists a constant C independent of K such that, for any $0 \le j \le m-1$ and $v \in W^m_p(K)$

   $$\sum_{|\alpha|=j} \|\partial^\alpha (v - Iv)\|_{L^{t_j}(\partial K)} \le C h_K^{m-j+(n-1)/t_j - n/p} |v|_{W^m_p(K)}.$$

2. For each edge e, denote ω_e be the union of the two triangles sharing e. Show that

   $$\|v\|^2_{L^2(e)} \le C \left(h_e^{-1} \|v\|^2_{L^2(\omega_e)} + h_e |v|^2_{H^1(\omega_e)} \right) \quad \forall v \in H^1(\omega_e).$$

3. Prove (5.9.1).