2021秋，有限元方法II，作业1

交作业时间：2021/10/15

The Mathematical Theory of Finite Element Methods:

- Chapter 0: 0.x.6, 0.x.11, 0.x.12, 0.x.13, 0.x.14
- Chapter 1: 1.x.5, 1.x.12, 1.x.42

Supplementary Questions:

1. For any $0 < s < 1$, show that

$$\int_{\mathbb{R}^n} \frac{|\mathrm{e}^{i\xi\cdot\eta} - 1|^2}{|\eta|^{n+2s}} \mathrm{d}\eta = \frac{1}{c(n,s)} |\xi|^{2s}.$$

   Find the positive constant $c(n,s)$.

2. Let Ω be a bounded Lipschitz domain. Show that

$$\|v\|_{L^2(\partial\Omega)} \leq C(\Omega) \left(\epsilon^{-1} \|v\|_{L^2(\Omega)} + \epsilon \|v\|_{H^1(\Omega)}\right),$$

   for any $v \in H^1(\Omega)$ and $\epsilon \in (0,1)$.

3. Let $\Omega \subset \mathbb{R}^n (n \geq 2)$ be a bounded Lipschitz domain. Show that

$$\|v\|_{L^{\frac{n}{n-1}}(\Omega)} \leq \frac{C(n)}{n-1} |v|_{W^{1,1}(\Omega)} \qquad \forall v \in W_0^{1,1}(\Omega).$$