有限元方法II，上机作业1

交作业时间：2019/10/23

要求:

- 用TeX写上机报告(中英文均可)，包含必要的数值结果讨论，页数上限12.
- 本次上机作业中，须自己组装刚度矩阵，推荐使用软件包iFEM.
- Problem 2 和 Problem 2'任选其一.
- 在课上提交上机报告打印版，截止时间前将程序和上机报告的源码发送至 snwu@math.pku.edu.cn

Consider the Poisson equation

$$\begin{cases} -\Delta u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g_D & \text{on } \Gamma_D, \\ \dfrac{\partial u}{\partial n} = g_N & \text{on } \Gamma_N = \partial\Omega \setminus \Gamma_D. \end{cases} \tag{1}$$

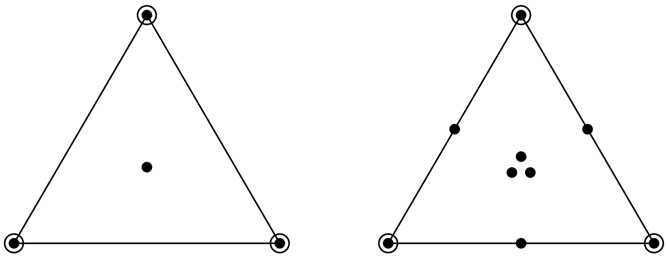
The computational domain is unit square, i.e., $\Omega = (0,1)^2$. Set $\Gamma_D = \{(0,y) \cup (1,y)\}$.

Problem 1. Using $\mathcal{P}_1$ Lagrange element to solve (1). Choose an exact solution u, then the source f and boundary data g_D, g_N can be obtained. Report the errors in H^1, L^2 and L^∞ norms.

Problem 2. Using $\mathcal{P}_3$ (and/or $\mathcal{P}_4$) Hermite element to solve (1). See Figure 1 for the plots of Hermite element. Report the errors in H^1, L^2 and L^∞ norms.

Problem 2'. Given $\Omega \subset \mathbb{R}^2$, consider the Allen-Cahn equation

$$\begin{cases} u_t - \Delta u + \dfrac{1}{\epsilon^2} f(u) = 0 & \text{in } \Omega_T := \Omega \times (0,T), \\ \dfrac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega_T := \partial\Omega \times (0,T), \end{cases} \tag{2}$$



(a) 2D Hermite element, $k = 3$

(b) 2D Hermite element, $k = 4$

Figure 1: Plots of 2D Hermite element.

with the initial condition $u|_{t=0} = u_0(x, y)$. Here, $f(u) = u^3 - u$. Given a time step size k ($k \leq \epsilon^2$), the fully implicit scheme to (2) is defined by seeking u^n for $n = 1, 2, \cdots$, such that

$$\frac{u^n}{k} - \Delta u^n + \frac{1}{\epsilon^2} f(u^n) = \frac{u^{n-1}}{k}. \tag{3}$$

Using $\mathcal{P}_1$ Lagrange element to solve (3) on each time step. Newton's method is suggested to solve the nonlinear problem. Set $\Omega = (-1, 1)^2$, $\epsilon = 0.02$. The numerical experiments are suggested to be taken with two different initial conditions:

(a) Circle, $T = 0.14$.

$$u_0(x, y) = \tanh\Big(\frac{\sqrt{x^2 + y^2} - 0.6}{\sqrt{2}\epsilon}\Big).$$

(b) Random initial condition: $u(x, y) \sim \mathcal{U}[-1, 1]$.