有限元方法II，作业3

交作业时间：2019/11/06

The Mathematical Theory of Finite Element Methods:

- Chapter 5: 5.x.10, 5.x.17, 5.x.21
- Chapter 8: 8.x.13, 8.x.19
- Chapter 9: 9.x.7, 9.x.19

Supplementary Questions:

1. Let V_h be a finite element space on a triangulation $\mathcal{T}_h$ of Ω. Define the norms on V_h as

$$\|w_h\|_{W_h^{2,p}(\Omega)}^p := \sum_{T\in\mathcal{T}_h} \|w_h\|_{W^{2,p}(T)}^p, \qquad \|w_h\|_{L_h^p(\Omega)} := \sup_{v_h\in V_h} \frac{(w_h, v_h)_\Omega}{\|v_h\|_{L^{p'}(\Omega)}},$$

where $1 < p < \infty$, and $1/p + 1/p' = 1$. Assume that a linear operator $\mathcal{L}_h : V_h \to V_h$ satisfies the following condition:

$$\|v_h\|_{L^{p'}(\Omega)} \lesssim \sup_{w_h\in V_h} \frac{(\mathcal{L}_h w_h, v_h)_\Omega}{\|w_h\|_{W_h^{2,p}(\Omega)}} \qquad \forall v_h \in V_h, h < h^*. \tag{1}$$

Show that for any $h < h^*$, the following stability holds:

$$\|w_h\|_{L^p(\Omega)} \lesssim \|\mathcal{L}_h w_h\|_{L_h^p(\Omega)} \qquad \forall w_h \in V_h.$$