2024春, 差分方法II，作业2

交作业时间：2024/03/22

Finite Difference Schemes and Partial Differential Equations:

- Chapter 10: 10.1.6, 10.1.7, 10.1.10, 10.3.4, 10.4.3

Finite Volume Methods for Hyperbolic Problems:

- Chapter 8: 8.3, 8.4

Supplementary Questions:

1. Let q, w be piecewise smooth weak solutions of scalar conservation law $q_t + f(q)_x = 0$, where f is convex. Assume that all the discontinuities are shocks. Let the nonoverlapping $I_k(t) := [x^k(t), x^{k+1}(t)]$ on which $q(x,t) - w(x,t)$ has sign $(-1)^k$. Show that for any k,

$$(-1)^k \left[f(w) - f(q) + (q-w)\frac{\mathrm{d}x}{\mathrm{d}t} \right] \Big|_{x^k}^{x^{k+1}} \leq 0.$$

2. Let $\boldsymbol{f}(\boldsymbol{z})$ be a smooth vector-valued function. Show that

$$\nabla_{\boldsymbol{z}} \boldsymbol{f} \text{ is symmetric} \iff \boldsymbol{f}^T = \nabla_{\boldsymbol{z}} r \quad \text{for some scalar-valued function } r(\boldsymbol{z}).$$