差分方法II，上机作业

截止时间：2023/06/16晚上12点

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限20.
- 程序语言不限, 但需要说明如何编译运行程序 (包含README文件或者在上机报告中说明).
- 截止时间前将程序源码和上机报告发送至snwu@math.pku.edu.cn

Two-scale method for solving the Monge-Ampère equation in 2D:

$$\begin{cases} \det D^2 u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g & \text{on } \partial\Omega. \end{cases}$$

The computational domain is unit square, i.e., $\Omega = (0,1)^2$. Let $\boldsymbol{x} = (x,y)^T$, $\boldsymbol{x}_0 = (0.5, 0.5)^T$. The numerical experiments are suggested to be taken with three different examples:

- Smooth and radial example:
$$u(\boldsymbol{x}) = \exp(|\boldsymbol{x}|^2/2), \quad f(\boldsymbol{x}) = (1+|\boldsymbol{x}|^2)\exp(|\boldsymbol{x}|^2).$$
- C^1 example:
$$u(\boldsymbol{x}) = \frac{1}{2}\left((|\boldsymbol{x}-\boldsymbol{x}_0| - 0.2)^+\right)^2, \quad f(\boldsymbol{x}) = \left(1 - \frac{0.2}{|\boldsymbol{x}-\boldsymbol{x}_0|}\right)^+.$$
- Twice differentiable in the interior domain, but has unbounded gradient near the boundary point $(1,1)$:
$$u(\boldsymbol{x}) = -\sqrt{2-|\boldsymbol{x}|^2}, \quad f(\boldsymbol{x}) = 2(2-|\boldsymbol{x}|^2)^{-2}.$$

The boundary data $g(\boldsymbol{x})$ can be obtained from the solution.

1. (90%) The Perron's iteration and Newton's method [1] are required to solve the nonlinear system from discretization. Choosing appropriate parameters $\delta = h^\alpha$, $\theta = h^\beta$ for some α, β, errors in L^∞ for different parameters should be reported and discussed.
2. (10%+bonus) Implement the filtered scheme [2] and compare the results for both smooth and C^1 cases.

References

[1] Ricardo H. Nochetto, Dimitrios Ntogkas, and Wujun Zhang. *Two-scale method for the Monge-Ampère equation: convergence to the viscosity solution*, **Mathematics of Computation**, 88(316), 637-664, 2019.

[2] Ricardo H. Nochetto, and Dimitrios Ntogkas. *Convergent two-scale filtered scheme for the Monge-Ampère equation*, **SIAM Journal on Scientific Computing**, 41(2), B295-B319, 2019.