

差分方法II，上机作业2

截止时间：2022/06/26晚上12点

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限15.
- 程序语言不限, 但需要说明如何编译运行程序 (包含README文件或者在上机报告中说明).
- 截止时间前将程序源码和上机报告发送至snwu@math.pku.edu.cn

Two-scale method for solving the Monge-Ampère equation in 2D:

$$\begin{cases} \det D^2 u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g & \text{on } \partial\Omega. \end{cases}$$

The computational domain is unit square, i.e., $\Omega = (0, 1)^2$. Let $\mathbf{x} = (x, y)^T$, $\mathbf{x}_0 = (0.5, 0.5)^T$. The numerical experiments are suggested to be taken with three different examples:

1. Smooth and radial example:

$$u(\mathbf{x}) = \exp(|\mathbf{x}|^2/2), \quad f(\mathbf{x}) = (1 + |\mathbf{x}|^2)\exp(|\mathbf{x}|^2).$$

2. C^1 example:

$$u(\mathbf{x}) = \frac{1}{2} ((|\mathbf{x} - \mathbf{x}_0| - 0.2)^+)^2, \quad f(\mathbf{x}) = \left(1 - \frac{0.2}{|\mathbf{x} - \mathbf{x}_0|} \right)^+.$$

3. Twice differentiable in the interior domain, but has unbounded gradient near the boundary point $(1, 1)$:

$$u(\mathbf{x}) = -\sqrt{2 - |\mathbf{x}|^2}, \quad f(\mathbf{x}) = 2(2 - |\mathbf{x}|^2)^{-2}.$$

The boundary data $g(\mathbf{x})$ can be obtained from the solution. The Perron's iteration or Newton's method can be used to solve the nonlinear system from discretization. Choosing appropriate parameters $\theta = h^\alpha$, $\theta = h^\beta$ for some α, β , errors in L^∞ for different parameters should be reported.