差分方法II，上机作业1

截止时间：2022/05/22晚上12点

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限15.
- 程序语言不限, 但需要说明如何编译运行程序 (包含README文件或者在上机报告中说明).
- 截止时间前将程序源码和上机报告发送至snwu@math.pku.edu.cn

Solving the conservation laws by using ENO (WENO) and SSPERK. The numerical fluxes should be taken as local Lax-Friedrichs (or Lax-Friedrichs), Roe and HLL (or HLLC, HLLE for Euler equations). Please report the following three numerical tests.

1. Solve the linear scalar advection equation to verify the convergence order.

2. One-dimensional Euler equations.

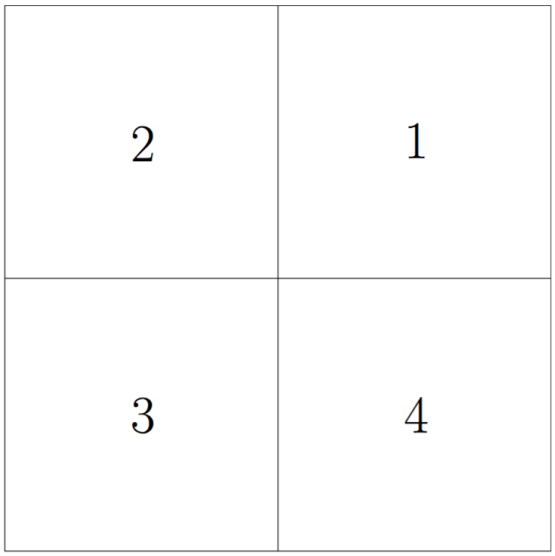
   Test problem: the Sod's problem defined by the initial conditions

$$\rho(x,0) = \begin{cases} 1 & x < 0.5, \\ 0.125 & x \geq 0.5, \end{cases}$$
$$\rho u(x,0) = 0,$$
$$E(x,0) = \frac{1}{\gamma - 1} \begin{cases} 1 & x < 0.5, \\ 0.1 & x \geq 0.5, \end{cases}$$

   where $\gamma = 1.4$. The computational domain is $[0,1]$, $T = 0.2$.

3. Two-dimensional Euler equations.

   Test problem: Configuration 4 of two-dimensional Riemann problem [1]. The computational domain is $[0,1]^2$, $\gamma = 1.4$, $T = 0.25$. The initial conditions are set constant in the four quadrants as sketched in Figure 1.



Configuration 4.

$$\overleftarrow{S_{21}} \qquad \overleftarrow{S_{32}} \qquad \overleftarrow{S_{41}} \qquad \overrightarrow{S_{34}}$$

The initial data are

$$\begin{array}{llll} p_2 = 0.35 & \rho_2 = 0.5065 & p_1 = 1.1 & \rho_1 = 1.1 \\ u_2 = 0.8939 & v_2 = 0 & u_1 = 0 & v_1 = 0 \\ \\ p_3 = 1.1 & \rho_3 = 1.1 & p_4 = 0.35 & \rho_4 = 0.5065 \\ u_3 = 0.8939 & v_3 = 0.8939 & u_4 = 0 & v_4 = 0.8939 \end{array}$$

Figure 1: Test problem for 2D Euler equations

References

[1] Kurganov, Alexander, and Eitan Tadmor. *Solution of twodimensional Riemann problems for gas dynamics without Riemann problem solvers*, **Numerical Methods for Partial Differential Equations**, 18(5), 584-608, 2002.