差分方法II，上机作业2

截止日期：2021/06/15晚上12点

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限15.
- 程序语言不限, 但需要说明如何编译运行程序 (包含README文件或者在上机报告中说明).
- 截止时间前将程序和上机报告的源码发送至 snwu@math.pku.edu.cn

The Lattice Basis Reduction method for the Monge-Ampère equation in 2D:

$$\begin{cases} \det D^2 u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g & \text{on } \partial\Omega. \end{cases}$$

The computation domain is $\Omega = (0,1)^2$. The numerical experiments are suggested to be taken with three different examples:

1. Smooth cone example:

$$u(\boldsymbol{x}) = \sqrt{\delta^2 + |\boldsymbol{x} - \boldsymbol{x}_0|^2},$$

   where $\boldsymbol{x}_0 = (0.5, 0.5)$ and $\delta = 0.1$.

2. Flat example:

$$u(\boldsymbol{x}) = (|\boldsymbol{x} - \boldsymbol{x}_0| - r_0)^2 + \frac{\varepsilon}{2}|\boldsymbol{x} - \boldsymbol{x}_0|^2,$$

   where $\boldsymbol{x}_0 = (0.5, 0.5)$, $r_0 = 0.2$ and $\varepsilon = 10^{-6}$.

3. Twice differentiable in the interior domain, but has unbounded gradient near the boundary point $(1, 1)$:

$$u(\boldsymbol{x}) = -\sqrt{2 - |\boldsymbol{x}|^2}, \quad f(\boldsymbol{x}) = 2(2 - |\boldsymbol{x}|^2)^{-2}.$$

The boundary data $g(\boldsymbol{x})$ can be obtained from the solution. Describe the method for solving the nonlinear system from discretization. Errors in L^∞ for different stencils should be reported.