差分方法II，上机作业1

截止日期：2021/05/21晚上12点

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限15.
- 程序语言不限, 但需要说明如何编译运行程序 (包含README文件或者在上机报告中说明).
- 截止时间前将程序和上机报告的源码发送至 snwu@math.pku.edu.cn

Consider the following Dirichlet problem:

$$\begin{cases} \Delta_p u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where

- $1 < p < \infty$: Δ_p represents the game theoretical p-Laplacian

$$\Delta_p = \frac{1}{p|\nabla u|^{p-2}} \mathrm{div}(|\nabla u|^{p-2}\nabla u);$$

- $p = \infty$, the infinity Laplacian is defined by

$$\Delta_\infty u := \frac{1}{|\nabla u|^2} \nabla u \otimes \nabla u : D^2 u.$$

Let $u(x)$ be a smooth function with non-vanishing gradient at x. Show that

$$\Delta_\infty u(x) = \min_{|y-x|=\delta} \frac{u(y)-u(x)}{\delta^2} + \max_{|y-x|=\delta} \frac{u(y)-u(x)}{\delta^2} + \mathcal{O}(\delta^2), \tag{2}$$

$$\Delta_p u = \frac{1}{p}\Delta u + \frac{(p-2)}{p}\Delta_\infty u. \tag{3}$$

Implement the wide stencil finite difference (WS-FD) method [1] for the (1) ($p \geq 2$). The explicit solution method and semi-implicit method should be used to solve the nonlinear system from discretization. Errors in L^∞ for different solutions, mesh sizes and stencils should be reported.

(**Optional**) Try the error estimate for WS-FD or Two-scale method.

References

[1] Adam M. Oberman. *Finite difference methods for the Infinity Laplace and p-Laplace equations*, **Journal of Computational and Applied Mathematics**, 245, 65-80, 2013.