

差分方法II, 作业2

交作业时间: 2021/04/26

Finite Difference Schemes and Partial Differential Equations:

- Chapter 8: 8.1.6, 8.2.5
- Chapter 9: 9.1.4, Example 9.2.2 (show the uniformly diagonalizable)
- Chapter 10: 10.1.6, 10.1.10, 10.3.4

1. Consider the elliptic problem in non-divergence form:

$$\mathbf{A}(x) : D^2u(x) = f(x), \quad \mathbf{A}(x) \geq 0.$$

If there exists $\Lambda_0 \geq \lambda_0 > 0$, such that $\lambda_0 \mathbf{I} \leq \mathbf{A}(x) \leq \Lambda_0 \mathbf{I}$ for all $x \in \Omega$, show that the non-divergence form is uniformly elliptic.

2. Given a symmetric matrix \mathbf{A} ($\mathbf{A} \neq 0$), show that $\gamma = \frac{\text{tr} \mathbf{A}}{\|\mathbf{A}\|_F^2}$ is the minimizer of

$$\min_{\tau \in \mathbb{R}} \|\tau \mathbf{A} - \mathbf{I}\|_F^2.$$

Here $\|\cdot\|_F$ represents the Frobenius norm. Find the minimum.