差分方法II，上机作业2

交作业时间：2020/06/24

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限12.
- 程序语言不限, 但需要说明如何编译运行程序 (包含README文件或者在上机报告中说明).
- 截止时间前将程序和上机报告的源码发送至 snwu@math.pku.edu.cn

The Oliker-Prussner method for the Monge-Ampère equation in 2D:

$$\begin{cases} \det D^2 u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g & \text{on } \partial\Omega. \end{cases}$$

The computation domain is $\Omega = (-1,1)^2$. The numerical experiments are suggested to be taken with three different examples:

1. Smooth solution:

$$u(\boldsymbol{x}) = \exp(|\boldsymbol{x}|^2/2), \quad f(\boldsymbol{x}) = (1+|\boldsymbol{x}|^2)\exp(|\boldsymbol{x}|^2).$$

2. Piecewise smooth solution:

$$u(\boldsymbol{x}) = \begin{cases} 2|\boldsymbol{x}|^2 & |\boldsymbol{x}| \le 1/2, \\ 2(|\boldsymbol{x}|-1/2)^2 + 2|\boldsymbol{x}|^2 & |\boldsymbol{x}| \ge 1/2, \end{cases}$$

$$f(\boldsymbol{x}) = \begin{cases} 16 & |\boldsymbol{x}| \le 1/2, \\ 64 - 16|\boldsymbol{x}|^{-1} & |\boldsymbol{x}| \ge 1/2. \end{cases}$$

3. Singular solution $u \in W_p^2$ with $p < 2$:

$$u(\boldsymbol{x}) = \begin{cases} x^4 + \frac{3}{2}x^{-2}y^2 & |y| \le |x|^3, \\ \frac{1}{2}x^2y^{2/3} + 2y^{4/3} & |y| > |x|^3, \end{cases}$$

$$f(\boldsymbol{x}) = \begin{cases} 36 - 9x^{-6}y^2 & |y| \le |x|^3, \\ \frac{8}{9} - \frac{5}{9}x^2y^{-2/3} & |y| > |x|^3. \end{cases}$$

The boundary data $g(\boldsymbol{x})$ can be obtained from the solution. Please report and discuss:

- The convergence history of the discrete Perron iteration;
- Errors in L^∞ and convergence order w.r.t. the mesh sizes;
- Errors and convergence order in the discrete W_p^2 norms. Here, the discrete W_p^2 norm is defined by

$$\|v\|_{W_p^2(\mathcal{N}_h^I)}^p = \sum_{j=1}^{4} \sum_{x_i \in \mathcal{N}_h^I} |\omega_i| \cdot |\Delta_{e_j} v(x_i)|^p,$$

where $e_1 = (1,0)$, $e_2 = (0,1)$, $e_3 = (1,1)$ and $e_4 = (1,-1)$ and Δ_{e_j} represents the central difference in the direction e:

$$\Delta_e v(x_i) := \frac{v(x_i + he) - 2v(x_i) + v(x_i - he)}{|e^2| h^2}.$$

ω_i represents the union of the elements that contain x_i.