差分方法II，上机作业1

交作业时间：2020/05/20

要求:

- 用TeX写上机报告(中英文均可), 包含必要的数值结果讨论, 页数上限12.
- 程序语言不限, 但需要说明如何编译运行程序 (包含README文件或者在上机报告中说明).
- 在课上提交上机报告打印版, 截止时间前将程序和上机报告的源码发送至 snwu@math.pku.edu.cn

Wide stencil finite difference method for the Monge-Ampère equation in 2D:

$$\begin{cases} \mathrm{det} D^2 u = f & \text{in } \Omega \subset \mathbb{R}^2, \\ u = g & \text{on } \partial\Omega. \end{cases}$$

The computational domain is unit square, i.e., $\Omega = (0,1)^2$. Let $\boldsymbol{x} = (x,y)^T$, $\boldsymbol{x}_0 = (0.5, 0.5)^T$. The numerical experiments are suggested to be taken with three different examples [1]:

1. Smooth and radial example:

$$u(\boldsymbol{x}) = \exp(|\boldsymbol{x}|^2/2), \quad f(\boldsymbol{x}) = (1+|\boldsymbol{x}|^2)\exp(|\boldsymbol{x}|^2).$$

2. C^1 example:

$$u(\boldsymbol{x}) = \frac{1}{2}\left((|\boldsymbol{x}-\boldsymbol{x}_0| - 0.2)^+\right)^2, \quad f(\boldsymbol{x}) = \left(1 - \frac{0.2}{|\boldsymbol{x}-\boldsymbol{x}_0|}\right)^+.$$

3. Twice differentiable in the interior domain, but has unbounded gradient near the boundary point $(1,1)$:

$$u(\boldsymbol{x}) = -\sqrt{2-|\boldsymbol{x}|^2}, \quad f(\boldsymbol{x}) = 2(2-|\boldsymbol{x}|^2)^{-2}.$$

The boundary data $g(\boldsymbol{x})$ can be obtained from the solution. The explicit solution method (required) and Newton's method (encouraged) can be used to solve the nonlinear system from discretization. Errors in L^∞ for different stencils should be reported.

References

[1] Brittany D. Froese, and Adam M. Oberman. *Convergent finite difference solvers for viscosity solutions of the elliptic Monge-Ampère equation in dimensions two and higher*, **SIAM Journal on Numerical Analysis**, 49(4), 1692-1714, 2011.