Points of infinite multiplicity of a planar Brownian motion.

Elie AÏDÉKON, *Paris 6, France*
Yueyun HU, *Paris 13, France*
Zhan SHI, *Paris 6, France*

Abstract: Points of infinite multiplicity are particular points which the Brownian motion visits infinitely often. Following a work of Bass, Burdzy and Khoshnevisan, we construct and study a measure carried by these points. Joint work with Yueyun Hu and Zhan Shi.

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Modelling Darwinian evolution as interacting particle systems

Gabriel BERZUNZA, *Georg-August-Universität Göttingen, Germany*
Anja STURM, *Georg-August-Universität Göttingen, Germany*
Anita WINTER, *Universität Duisburg-Essen, Germany*

Abstract: The study of interactions between organisms and their environment that influence their performance, reproductive success, and contribute to phenotype variation, i.e. Darwinian evolution, it is a major problem in evolutionary ecology and population genetics. In this talk, we are interested in modelling the dynamic of populations as a Markov point process whose generator captures the dynamics over continuous time of a branching process with mutation which behavior may depend on each individuals trait and the iterations between them, namely the competition between individuals for limited resources. Traits are hereditarily transmitted from a parent to its offspring unless a mutation occurs. In this case, the offspring makes an instantaneous mutation step at birth to new trait values. We are interested in generalize previous microscopic point of views by allowing individuals to have multiple offspring at the reproduction event. Moreover, the reproduction law may also be influenced by the phenotype variation due to mutation. In the context of ecology most of the models that have been used for the study of cell division rely on binary fission. However, some organisms follow alternative reproductive strategies in order to remain competitive and propagate, which include multiple offspring mechanisms. By combining various scalings on population size, birth and death rates, mutation rate, mutation step, we establish a superprocess limit for the interacting particle system described previously. This new superprocess appears as a generalization of the existing models for spatially structured populations.

Universal vanishing corections on the position of fronts in the Fisher-KPP class

Eric BRUNET, *LPS, ENS Paris, France*

Abstract: The distribution function of the rightmost particle in a branching Brownian motion satisfies the Fisher-KPP equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u - u^2$$

Such an equation appears also in biology, chemistry or theoretical physics to describe a moving interface, or a front, between a stable and an unstable medium.

Thirty years ago, Bramson gave rigorous sharp estimates on the position of the front, and, fifteen years ago, Ebert and van Saarloos heuristically identified universal vanishing corrections.

In this presentation, I will present a novel way to study the position of such a front, which allows to recover all the known terms and find some new ones. We start by studying a front equation where the non-linearity is replaced by a condition at a free boundary, and we show how to extend our results to the actual Fisher-KPP.

A distribution equation and its application on tail baheviour of some multi-type Galton-Watson tree

Xinxin CHEN, *Institut Camille Jordan, University Lyon 1, France*

Abstract: We consider a biased random walk on Galton-Watson tree whose potentiels are given by some branching random walk. In the current case its edge local times form a multi-type Galton-Watson tree. The maximal edge local time satisfies a distribution equation whose initial version was considered by Bertoin [1]. We get the tail distribution of this maximum by studying this distribution equation and the tail behaviours of the associated branching random walk. This talk is on one joint work with C. Ma and another joint work with L. de Raphélis.

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Some properties of a max-type recursive model

Xinxing CHEN, *Shanghai Jiaotong University, China*

Abstract: We consider a simple max-type recursive model which was introduced in the study of depinning transition in presence of strong disorder by Derrida and Retaux. Our interest is focused on the critical regime, for which we study the extinction probability and the moment generating function. This talk is based on a joint work with Bernard Derrida, Yueyun Hu, Mikhail Lifshits and Zhan Shi.

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Entrance and exit at infinity for stable SDEs

Leif DOERING, *University of Mannheim, Germany*

Abstract: Most of us have heard of (or know well) the boundary classification of one-dimensional SDEs. For instance, Feller's test for explosion gives necessary and sufficient conditions on the drift and volatility to allow finite time explosion. Other tests give precise conditions if solutions can start from a boundary point or reach in finite time a finite boundary point. Now replace the driving Brownian motion by a Lévy process, say a stable one. Do you know how to generalize Feller's results? Me neither, but at least for the special case without drift and infinite boundary points we can give a full characterization of finite time explosion and entrance from infinity (also known as coming down from ininity). This is joint work with Andreas Kyprianou.

Limits of multiplicative inhomogeneous random graphs and Lévy trees

Nicolas BROUTIN, *Sorbone Université, France*
Thomas DUQUESNE, *Sorbone Université, France*
Minmin WANG, *University of Bath, United Kingdom*

Abstract: We consider a model of inhomogeneous random graphs that extend Erdős-Rényi graphs and that shares a close connection with the multiplicative coalescence, as pointed out by Aldous [*Ann. Probab.,* vol. 25, pp. 812-854, 1997]. Theses models have been studied first by Aldous and Limic [*Electron. J. Probab.,* vol. 3, pp. 1-59, 1998] and their connected components evolves as a multiplicative coalescent: namely, let N be the number of vertices and let $w_1, \ldots, w_N$ be a set of positive weights; we independently put an edge between vertices i and j with probability $p_{ij} = 1 - e^{-w_i w_j / s}$ (in our case, we consider, $s = w_1 + \ldots + w_N$).

Our results are the following: we first generate such graphs by an exploration that reduces to a LIFO queue. This point of view allows to code an appropriate spanning tree of the graph thanks to a contour process (and a modified Lukasiewicz path) and to get a simple control on the surplus edges. The spanning tree encompasses most of the metric structure. This construction also allows to embed such graphs into Galton-Watson trees.

This embedding transfers asymptotically into an embedding of the limit objects into a forest of Lévy trees, which allows us to prove a limit theorem and an explicit construction of the limit objects from the excursions of a Lévy-type process. As a consequence of our construction, we give transparent and explicit condition for the compactness of the limit objects and determine their fractal dimensions. These results extend and complement several previous results that had obtained via model- or regime-specific proofs, for instance: the case of Erdős-Rényi random graphs obtained by Addario-Berry, Goldschmidt and B. [*Probab. Theory Rel. Fields,* vol. 153, pp. 367-406, 2012], the *asymptotic homogeneous* case as studied by Bhamidi, Sen and Wang [*Probab Theory Rel. Fields,* vol. 169, pp. 565-641, 2017], or the power-law case as considered by Bhamidi, Sen and van der Hofstad [*Probab. Theory Rel. Fields,* vol. 170, pp. 387-474, 2018].

Liouville measure as a multiplicative cascade via local sets of the Gaussian free field

Juhan ARU, *ETH Zürich, Switzerland*
Ellen POWELL, *ETH Zürich, Switzerland*
Avelio SEPÚLVEDA, *Université Lyon 1, France*

Abstract: Gaussian multiplicative chaos (GMC) theory, initiated by Kahane ([7]) in the 80s as a generalization of multiplicative cascades, aims to give meaning to "exp(Γ)" for rough Gaussian fields Γ. One of the cases of interest, due to its connection with so-called "Liouville quantum gravity" and the KPZ relations ([6]), is when the underlying field Γ is a multiple γ of the planar Gaussian free field (GFF). In this case one can construct the associated GMC measures for $\gamma \in [0, 2)$, the subcritical regime ([6]), and $\gamma = 2$, the critical regime ([5]). We call these the "Liouville measures". We provide new constructions of these measures ([2]) using a coupling between the GFF and the conformal loop ensemble CLE_4, in which the loops of this ensemble play the role of local (or stopping) sets for the GFF ([4],[9]). As a special case we recover E. Aïdekon's construction ([1]) of random measures using nested conformally invariant loop ensembles, and verify his conjecture that certain CLE_4 based limiting measures are equal in law to the Liouville measures. One of the aims of our construction is to build a link between the theory of branching random walks (or multiplicative cascades) and GMC theory, which may allow one to transfer results and techniques from the former to the latter. As an example, using this new construction together with the work of T. Madaule for the branching random walk ([8]), we are able to prove in ([3]) that the critical Liouville measure ($\gamma = 2$) can be obtained as a limit of the (suitably rescaled) subcritical measures.

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Measure-Valued Processes and Random Measures

Shui FENG, *McMaster University, Canada*

Abstract: The focus of the talk is on several random measures and the corresponding measure-valued processes.The random measures include the Poisson-Dirichlet distribution, the GEM distribution , the Dirichlet process, and their two-parameter generalizations. The talk will start with a survey of existing results followed by a discussion of my recent joint work with Wei Sun. Several open issues will be touched along the way.

Soliton decomposition of the Box Ball System

Chi NGUYEN, *Universidad de Buenos Aires, Argentina*
Leonardo ROLLA, *Conicet, Argentina and NYU Shanghai, China*
Minmin WANG, *University of Bath, UK*
Davide GABRIELLI, *Università de L'Aquila, Italy*
Pablo A. FERRARI, *Universidad de Buenos Aires, Argentina*

Abstract: The Box-Ball System (BBS) is a cellular automaton introduced by Takahashi and Satsuma (TS) as a discrete counterpart of the Korteweg & de Vries (KdV) differential equation. Both systems exhibit solitons, solitary waves that conserve shape and speed even after collision with other solitons. The BBS has state space $\{0,1\}^Z$ representing boxes which may contain one ball or be empty. A carrier visits successively boxes from left to right, picking balls from occupied boxes and deposing one ball, if carried, at each visited empty box. Building on the TS identification of solitons, we provide a soliton decomposition of the ball configurations, show that the dynamics reduces to a hierarchical translation of the components and prove that shift stationary measures with independent soliton components are invariant for the dynamics. We also prove that the asymptotic speed of a tagged soliton of size k converges to a positive real number v_k and exhibit the equations satisfied by the speeds $(v_k)_{k\geq 1}$. A detailed analysis shows that among many others, product measures and Ising measures have independent soliton components.

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The problem with genealogies...

Simon HARRIS *University of Auckland, New Zealand*

Abstract: We will discuss a problem (or possibly two) relating to genealogies in a branching process.

A time-change for a stationary branching process

Romain ABRAHAM, *MAPMO, Université d'Orléans, France*
Jean-François DELMAS, *CERMICS, Université Paris-Est, France*
Hui HE, *Beijing Normal University, China,*

Abstract: We consider a stationary branching process obtained by a subcritical continuous state branching process with immigration. We shall show that the reduced tree of the process, after a time-change, is a continuous time Galton-Watson tree with immigration. As a by-product, we get that in the stable setting the sizes of all families at any time induce a Poisson-Kingman partition, which forms a Poisson-Dirichlet distribution in the quadratic case.

A STOCHASTIC EQUATION FOR THE HEIGHT PROCESS OF A POPULATION WITH COMPETITION

Zenghu LI, *Beijing Normal University, China*
Étienne PARDOUX, *Aix-Marseille Université, France*
Anton WAKOLBINGER, *Goethe-Universität, Germany*

Abstract: A continuous-state branching process is the mathematical model for the random evolution of a large population. The genealogical structure the population is represented by a Lévy forest, which is uniquely characterized by its height process. The later was constructed by Le Gall and Le Jan (1998) and Duquesne and Le Gall (2002) as a functional of a spectrally positive Lévy process. A flow of continuous-state branching processes was constructed in Dawson and Li (2012) as strong solutions to a stochastic equation driven by space-time noises. By a simple variation of the stochastic equation, a more general population model can be constructed by introducing a competition structure through a function called the competition mechanism. For a diffusion model with logistic computation, the genealogical structures were characterized by Le et al. (2013) and Pardoux and Wakolbinger (2011) in terms of a stochastic equation of the corresponding height process. The genealogical forest of the general model with competition was constructed in the recent work of Berestycki et al. (2017+) by pruning the Lévy forest according to an intensity identified as a fixed point of certain transformation on the space of all adapted intensities determined by the competition mechanism. In this talk, we present a construction of the corresponding height process in terms of a stochastic integral equation based on a Poisson point measure. This generalizes the results of Le et al. (2013) and Pardoux and Wakolbinger (2011) to general branching mechanisms. The advantage of this construction is that it unifies the treatments for models with or without competition. However, up to now the stochastic equation is established only for the model with a nontrivial diffusion component.

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Dynamical freezing in a spin-glass with logarithmic correlations

Oren LOUIDOR, *Technion, Israel*

Abstract: We consider a continuous time random walk on the 2D torus, governed by the exponential of the discrete Gaussian free field acting as potential. This process can be viewed as Glauber dynamics for a spin-glass system with logarithmic correlations. Taking temperature to be below the freezing point, we then study this process both at pre-equilibrium and in-equilibrium time scales. In the former case, we show that the system exhibits aging and recover the arcsine law as asymptotics for a natural two point temporal correlation function. In the latter case, we show that the dynamics admits a functional scaling limit, with the limit given by a variant of Kolmogorov's K-process, driven by the limiting extremal process of the field, or alternatively, by a super-critical Liouville Brownian motion. Time permitting, I will talk only on one or both cases. Joint work with A. Cortines, J. Gold and A. Svejda.

Extremal behavior of continuous-state branching processes

Chunhua MA, *Nankai University, China*
Wei XU, *Humboldt-Universität zu Berlin, Germany*

Abstract: We study the extremal behavior of a continuous-state branching process (CSBP) driven by a Lévy process that is regularly varying with index $\alpha > 1$. We show that in the subcritical case the extremal behavior of the CSBP is due to one big jump of the driving Lévy process and its limit measure is given associated with regular variation on the space of càdlàg functions, while in the critical case multiple big jumps are required to make the extreme event happen. In addition, along the similar line, we also study the conditioned asymptotic behaviors of regularly varying Lévy processes with negative drift, and based on it, the asymptotics of the expectation of some exponential funcitonals of such Lévy processes are derived.

A time-change for a stationary branching process

Pascal MAILLARD, *Maître de Conférences, Université Paris-Sud, France*

Abstract: I will present recent results (obtained with Michel Pain) on the fluctuations of the Gibbs measure of branching Brownian motion at the critical temperature. By Gibbs measure I mean here the measure whose atoms are the positions of the particles, weighted by their Gibbs weight. It is known that this Gibbs measure, after a suitable scaling, converges in probability to a marginal of a Brownian meander times the derivative martingale limit. We prove a non-standard central limit theorem for the integral of a function against the Gibbs measure, for a large class of functions. The possible limits are 1-stable laws with arbitrary asymmetry parameter depending on the function. In particular, the derivative martingale and the usual additive martingale satisfy such a central limit theorem with, respectively, a totally asymmetric and a Cauchy distribution.

Decompositions of multitype continuous-state branching processes

Sandra PALAU, *University of Bath, UK*

Abstract: We can construct different decomposition for multi-type continuous-state branching processes with general branching mechanism. In this talk we are going to focus in the backbone and in the spine decomposition. We will show the similarities and differences of them. Finally, we are going to provide some applications. This work is based in the papers [1] and [2].

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Extinction rate of continuous state branching processes in critical Lévy environments

Vincent BANSAYE, *CMAP, Ecole Polytechnique, France*
Juan Carlos PARDO, *Centro de Investigación en Matemáticas, Mexico*
Charline SMADI, *IRSTEA, France*

Abstract: We determine the speed of extinction of continuous state branching processes (CSBPs) in an oscillating Lévy environment satisfying Spitzer's condition. Our approach relies on the path analysis of the process together with its environment, conditionally on the survival event, following similar reasonings to the discrete setting of branching processes in iid random environment of Afanasyev et al. [1]. In particular, we characterize a CSBP in a Lévy environment conditioned to stay positive as a weak solution of a stochastic differential equation and determine its long time behaviour.

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Branching Brownian motion with decay of mass and the non-local Fisher-KPP equation

Louigi Addario-Berry *McGill University, Canada*
Sarah PENINGTON *University of Oxford, UK*
Julien Berestycki *University of Oxford, UK*

Abstract: We add a competitive interaction between nearby particles in a branching Brownian motion (BBM). Each particle has a mass, which decays at rate proportional to the mass density in a window centred at the location of the particle. The total mass of the system increases through branching events. In standard BBM, we may define the front location at time t as the location of the rightmost particle. For BBM with decay of mass, it makes sense to instead define the front location as the location at which the local mass density drops from $\Theta(1)$ to $o(1)$. We can show that in a weak sense this front is $\Theta(t^{1/3})$ behind the front for standard BBM.

We can also show that at large times, over a bounded time interval, the local mass density for BBM with decay of mass is well approximated by the solution of a PDE known as the non-local Fisher-KPP equation. This relationship between the particle system and the PDE allows us to control the behaviour of the local mass density behind the front at large times, and also to use intuition from the particle system setting to prove new results about the PDE.

Several interesting questions about this model remain open.

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Ornstein-Uhlenbeck type growth-fragmentation processes

Quan SHI, *University of Oxford, UK*

Abstract: Growth-fragmentation processes describe systems of particles in which each particle may grow larger or smaller, and divide into smaller ones as time proceeds. Previous studies on growth-fragmentations mainly focus on the self-similar case, whereas we introduce a new family of growth-fragmentation that is closely related to Lévy driven Ornstein-Uhlenbeck type processes. We establish a convergence criterion for a sequence of such growth-fragmentations. We show that, under certain conditions, this system possesses a law of large numbers.

Proper normalization and non-degenerate strong limit for supercrirical superprocesses

Yan-Xia REN, *Peking University, China*
Renming SONG, *University of Illinois, USA*
Rui ZHANG, *Capital Normal University, China*

Abstract: Suppose that $X = \{X_t, t \geq 0; \mathbf{P}_\mu\}$ is a supercritical superprocess in a locally compact separable metric space E. Let ϕ_0 be a positive eigenfunction corresponding to the first eigenvalue λ_0 of the generator of the mean semigroup of X. Then $M_t := e^{-\lambda_0 t}\langle \phi_0, X_t\rangle$ is a positive martingale. Let M_∞ be the limit of M_t. It is known (see [1]) that M_∞ is nondegenerate iff the $L \log L$ condition is satisfied. In this paper we deal with the case when the $L \log L$ condition is not satisfied. We prove that there exist a function γ_t on $[0, \infty)$ and a non-degenerate random variable W such that for any finite nonzero Borel measure μ on E,

$$\lim_{t\to\infty} \gamma_t \langle \phi_0, X_t\rangle = W, \qquad \text{a.s.-}\mathbf{P}_\mu.$$

We prove that W has strictly positive density on $(0, \infty)$. We also investigate the small value probability and tail probability problems of W.

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REDUCED CRITICAL PROCESSES FOR SMALL POPULATIONS

Minzhi LIU, *Beijing Normal University, China*
Vladimir VATUTIN, *Steklov Mathematical Institute, Russia*

Abstract: Let $\{Z(n), n \geq 0\}$ be a Galton-Watson branching process with $Z(0) = 1$ in which particles produce children in accordance with probability generating function

$$f(s) = \mathbf{E}s^{\xi} = \sum_{k=0}^{\infty} f_k s^k, \qquad \text{g.c.d.}\ \{k : f_k > 0\} = 1,$$

and let $Z(m, n)$ be the number of particles in the process at moment $m < n$ having a positive number of descendants at moment n.

It is known that if $\mathbf{E}\xi = 1$, $2B := Var\xi \in (0, \infty)$ then

- $$\lim_{n\to\infty} \mathbf{P}\left(\frac{Z(n)}{Bn} \leq x | Z(n) > 0\right) = 1 - e^{-x}, 0 \leq x < \infty,$$

  meaning that, given survival of the process to moment n, the number of particles at this moment is, as a rule, proportional to n.

- The process $\{Z(nt, n), 0 \leq t \leq 1 | Z(n) > 0\}$ weakly converges, as $n \to \infty$ to the Yule process on $[0, 1]$.

We consider a kind of large deviation phenomenon for the reduced process. Namely, we show that if a function $\varphi(n) = o(n)$ as $n \to \infty$ then, under the conditions above, for any $x \in (0, \infty)$

$$\lim_{n\to\infty} \mathbf{E}\left[s^{Z(n - x\varphi(n), n)} \big| 0 < Z(n) \leq B\varphi(n)\right] = sx\frac{1 - e^{-(1-s)/x}}{1 - s}.$$

Besides, for any fixed $t \in [0, 1)$ and any $a \in (0, \infty]$

$$\lim_{n\to\infty} \mathbf{E}\left[s^{Z(nt,n)} \big| 0 < Z(n) \leq aBn\right] = s\frac{1 - t}{1 - ts}\frac{1 - e^{-(1-ts)a/(1-t)}}{1 - e^{-a}}.$$

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A probabilistic approach to spectral analysis of growth-fragmentation equations

Alexander R. WATSON, *University of Manchester, UK*

Abstract: The growth-fragmentation equation describes a system of growing and dividing particles, and arises in models of cell division, protein polymerisation and even telecommunications protocols. Several important questions about the equation concern the asymptotic behaviour of solutions at large times: at what rate do they converge to zero or infinity, and what does the asymptotic profile of the solutions look like? Does the rescaled solution converge to its asymptotic profile at an exponential speed? These questions have traditionally been studied using analytic techniques such as entropy methods or splitting of operators. In this talk, I discuss a probabilistic approach to the study of this asymptotic behaviour. The method is based on the Feynman-Kac formula and a related martingale technique. This is joint work with Jean Bertoin.

Gaussian Free Field: Level Lines and Connection Probabilities

Hao WU, *Tsinghua University, China*

Abstract: Gaussian free field (GFF) is of great interest in statistical physics models. On the one hand, it is the height function of many discrete lattice models, for instance dimer model; on the other hand, it is the building block in many constructions in quantum field theory. In this talk, we will discuss the level lines of GFF and we will focus on the following topics: [1] The level lines of GFF are SLE(4). [2] The connection probabilities of level lines of GFF are encoded by the collection of pure partition functions associated to multiple SLE(4)s. [3] Existence and uniqueness of pure partition functions associated to multiple SLE(4)s. Indeed, the pure partition functions associated to multiple SLEs can be generalized to all $\kappa \in (0, 8)$, and it is natural to ask for the connection probabilities for other lattice models. We will discuss the known result on loop- erased random walk and the open question about the crossing probabilities for the critical Ising interfaces.

Uniqueness problems for SPDEs and SDEs from population models

Jie XIONG, *Southern University of Science & Technology, Shenzhen, China*

Abstract: In this talk I will present a few open problems about uniqueness problems for some SPDEs or SDEs arising from catalytic super-Brownian motions and its simplified forms.

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law of large numbers for supercritical superprocesses with non-local branching

Sandra PALAU, *Department of Mathematical Sciences, University of Bath, UK.*
Ting YANG, *School of Mathematics and Statistics, Beijing Institute of Technology, China*

Abstract: In this talk, we consider the superprocesses with general branching mechanisms. Under mild conditions, there exists a leading eigenvalue of the associated Schrödinger operator. The sign of this eigenvalue distinguishes between the cases where there is local extinction and exponential growth. When this leading eigenvalue is positive (supercritical), we establish the weak and strong law of large numbers for the superprocesses.

Williams decomposition for superprocesses and the behavior near the extinction

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Abstract: We decompose the genealogy of a general superprocess with spatially dependent branching mechanism with respect to the last individual alive (Williams decomposition). This is a generalization of the main result of Delmas and Hénard (*Electron. J. Probab.*, 2013) where only superprocesses with spatially dependent quadratic branching mechanism were considered. As an application of the Williams decomposition, we prove that, for some superprocesses, the normalized total measure will converge to a point measure at its extinction time. This partially generalizes a result of Tribe(*Ann. Probab.*, 1992)in the sense that our branching mechanism is more general.

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Speeds of coming down from infinity for continuous-state nonlinear branching processes

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Abstract: We consider a class of nonlinear continuous-state branching processes which can be obtained from spectrally positive Lévy processes via Lamperti type time transform. Intuitively, they are the branching processes whose branching rates depend on the current population sizes. The extinction, explosion and coming down from infinity behaviors for such processes have been studied in Li (2016) and Li et al. (2017).

In this talk we further discuss the small time asymptotic behaviors of the processes. By analyzing Laplace transforms of weighted occupation times and fluctuation behaviors for spectrally positive Lévy processes, we solve a one-sided exit problem for the nonlinear branching processes and identify the speeds of coming down from infinity in different scenarios.

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