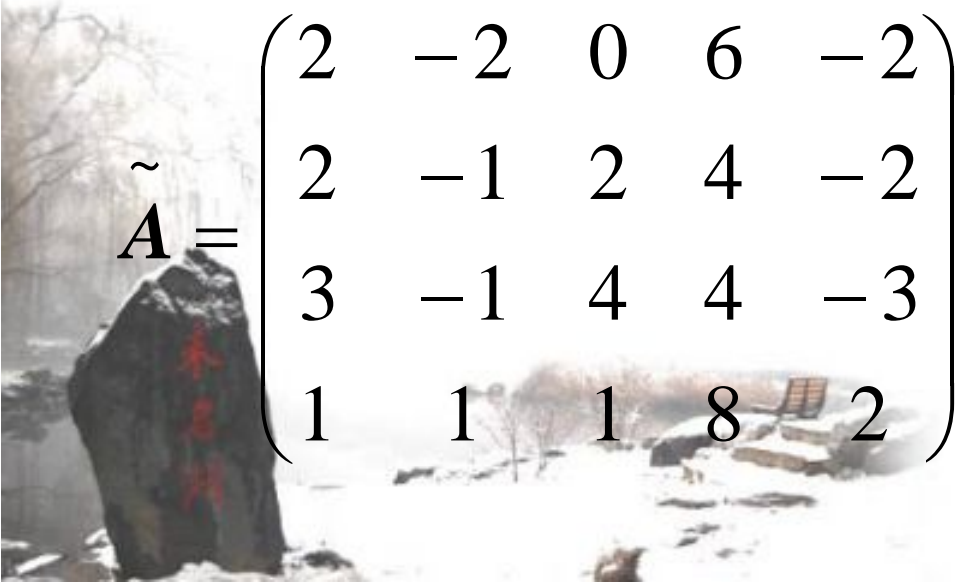


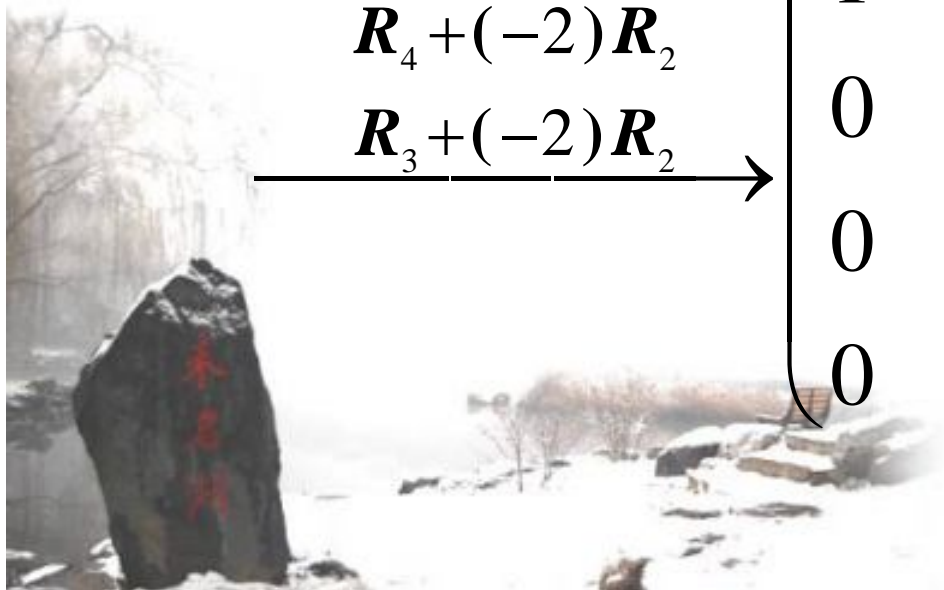
Gauss消元法

$$\begin{cases} 2x_1 & -2x_2 & & +6x_4 & = -2 \\ 2x_1 & -x_2 & +2x_3 & +4x_4 & = -2 \\ 3x_1 & -x_2 & +4x_3 & +4x_4 & = -3 \\ x_1 & +x_2 & +x_3 & +8x_4 & = 2 \end{cases}$$


$$\tilde{\mathbf{A}} = \begin{pmatrix} 2 & -2 & 0 & 6 & -2 \\ 2 & -1 & 2 & 4 & -2 \\ 3 & -1 & 4 & 4 & -3 \\ 1 & 1 & 1 & 8 & 2 \end{pmatrix} \xrightarrow{\frac{1}{2}\mathbf{R}_1} \begin{pmatrix} 1 & -1 & 0 & 3 & -1 \\ 2 & -1 & 2 & 4 & -2 \\ 3 & -1 & 4 & 4 & -3 \\ 1 & 1 & 1 & 8 & 2 \end{pmatrix}$$

$$\begin{array}{l} \mathbf{R}_4 + (-1)\mathbf{R}_1 \\ \mathbf{R}_3 + (-3)\mathbf{R}_1 \\ \mathbf{R}_2 + (-2)\mathbf{R}_1 \end{array} \rightarrow \begin{pmatrix} 1 & -1 & 0 & 3 & -1 \\ 0 & 1 & 2 & -2 & 0 \\ 0 & 2 & 4 & -5 & 0 \\ 0 & 2 & 1 & 5 & 3 \end{pmatrix}$$

$$\begin{array}{l} \mathbf{R}_4 + (-2)\mathbf{R}_2 \\ \mathbf{R}_3 + (-2)\mathbf{R}_2 \end{array} \rightarrow \begin{pmatrix} 1 & -1 & 0 & 3 & -1 \\ 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -3 & 9 & 3 \end{pmatrix}$$



$$\xrightarrow{R_{34}} \begin{pmatrix} 1 & -1 & 0 & 3 & -1 \\ 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & -3 & 9 & 3 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$



LU分解

$$\mathbf{L}_k = \mathbf{I} - l_k \mathbf{e}_k^T$$

$$l_k = (0, \dots, 0, l_{k+1,k}, \dots, l_{nk})^T$$

$$\mathbf{L}_k^{-1} = \mathbf{I} + l_k \mathbf{e}_k^T$$



$$\begin{array}{c}
 \begin{bmatrix}
 \times & \times & \times & \times \\
 \times & \times & \times & \times \\
 \times & \times & \times & \times \\
 \times & \times & \times & \times
 \end{bmatrix} \\
 A
 \end{array}
 \xrightarrow{L_1}
 \begin{array}{c}
 \begin{bmatrix}
 \times & \times & \times & \times \\
 0 & \times & \times & \times \\
 0 & \times & \times & \times \\
 0 & \times & \times & \times
 \end{bmatrix} \\
 L_1 A
 \end{array}
 \xrightarrow{L_2}
 \begin{array}{c}
 \begin{bmatrix}
 \times & \times & \times & \times \\
 & \times & \times & \times \\
 & 0 & \times & \times \\
 & 0 & \times & \times
 \end{bmatrix} \\
 L_2 L_1 A
 \end{array}
 \xrightarrow{L_3}
 \begin{array}{c}
 \begin{bmatrix}
 \times & \times & \times & \times \\
 & \times & \times & \times \\
 & & \times & \times \\
 & & 0 & \times
 \end{bmatrix} \\
 L_3 L_2 L_1 A
 \end{array}
 \end{array}$$



算法

for k = 1:n-1

$$A(k+1:n,k) = A(k+1:n,k)/A(k,k)$$

$$A(k+1:n,k+1:n) = A(k+1:n,k+1:n) - \\ A(k+1:n,k) * A(k,k+1:n)$$

end

