Review Summary for Numerical PDE

1. Basic Contents : (1) FDM for Elliptic Equations; (2) FDM for Parabolic Equations; (3) FDM for Hyperbolic Equations; (4) FEM for Elliptic Equations.

2. Basic Concepts:

(1) Explicit and implicit finite difference schemes, grid ratio and mesh refinement path;

(2) Truncation error, consistency, stability, convergence, order of approximation and convergence of a finite difference scheme;

(3) Characteristics and Riemann invariants of hyperbolic equations, domain of dependence and CFL condition;

(4) Amplification factor and matrix of a finite difference scheme, amplitude error and phase error, dissipation and dispersion, group speed of finite difference solutions;

(5) Weak solutions of hyperbolic conservation laws and shock speed;

(6) Conservation and conservative finite difference schemes, finite volume methods;

(7) Fourier analysis and L^2 stability, maximum principle and L^∞ stability, practical and strong stability, Pèclet number of a grid;

(8) Variational forms and weak solutions of boundary value problems of elliptic equations, coercive and natural boundary conditions;

(9) Galerkin method and Ritz method, conforming and nonconforming finite element methods;

(10) Finite element and finite element function spaces, finite element interpolation operators and functions, finite element equations and finite element solutions;

(11) Affine equivalent and iso-parametric equivalent of finite elements, regular and quasi-uniform finite element triangulations;

3. Basic Methods and Techniques:

(1) Comparison function and error analysis based on maximum principle;

(2) Taylor expansion, Fourier analysis and maximum principle;

(3) Methods of modified equation analysis and energy analysis;

(4) Making best use of conservation and characteristics of a solution, proper treatments of initial and boundary conditions;

(5) Sobolev spaces and their basic properties, such as embedding theorem, trace theorem, Poincaré-Friedrichs inequality, etc..

(6) Affine equivalent, polynomial invariant operators, quotient spaces and quotient norms, the relations between Sobolev semi-norms on affine equivalent open sets;

(7) Aubin-Nische technique, Bramble-Hilbert lemma, bilinear lemma;

4. Basic schemes:

(1) Five point finite difference scheme for 2-dimentsional Poisson equation;

(2) Explicit, implicit, Crank-Nicolson and θ- schemes of parabolic equations;

(3) Schemes with heat conservation property;

(4) ADI and LOD schemes;

(5) Upwind, Lax-Wendroff, Box and leap-frog schemes;

(6) Conservative schemes and finite volume schemes;

(7) Class C^0 conforming finite element methods for second order elliptic equations.

5. Basic theorems and conclusions:

(1) The maximum principle and the comparison theorems;

(2) Lax equivalence theorem;

(3) CFL condition, von Neumann condition, practical stability and strong stability conditions;

(4) Lax-Milgram lemma, Céa lemma, the first and second strang lemma;

(5) The a priori error estimates and convergence of finite element solutions of elliptic equations;