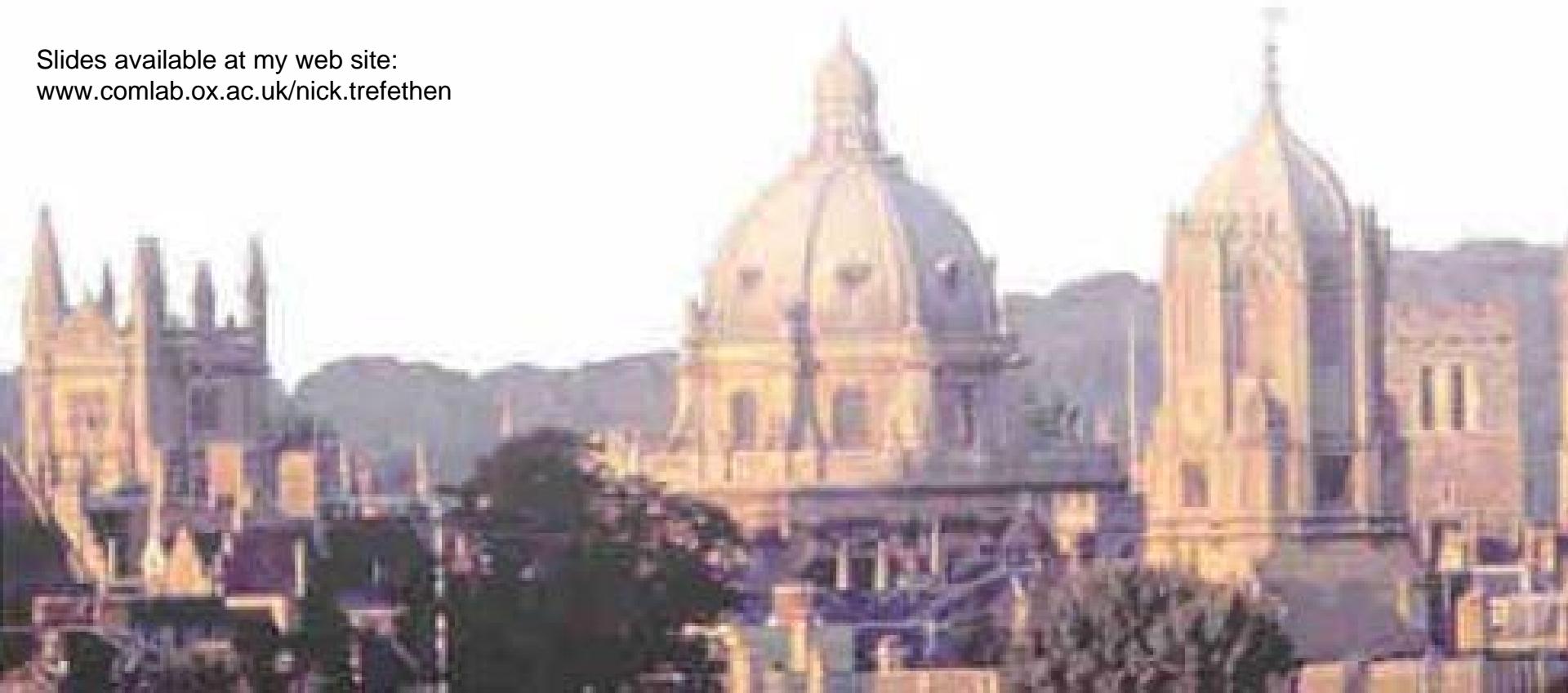


Nick Trefethen  
Oxford Computing Lab

# Who invented the great numerical algorithms?

Slides available at my web site:  
[www.comlab.ox.ac.uk/nick.trefethen](http://www.comlab.ox.ac.uk/nick.trefethen)



A discussion over coffee.  
Ivory tower or coal face?

# **SOME MAJOR DEVELOPMENTS IN SCIENTIFIC COMPUTING**

(29 of them)

## **Before 1940**

Newton's method  
least-squares fitting  
Gaussian elimination  
Gauss quadrature  
Adams formulae  
Runge-Kutta formulae  
finite differences

## **1940-1970**

floating-point arithmetic  
splines  
Monte Carlo methods  
simplex algorithm  
conjugate gradients & Lanczos  
Fortran  
stiff ODE solvers  
finite elements

orthogonal linear algebra  
QR algorithm  
Fast Fourier Transform  
quasi-Newton iterations

## **1970-2000**

preconditioning  
spectral methods  
MATLAB  
multigrid methods  
IEEE arithmetic  
nonsymmetric Krylov iterations  
interior point methods  
fast multipole methods  
wavelets  
automatic differentiation

Before 1940

# Newton's Method for nonlinear eqs.

Heron, al-Tusi 12c, Al Kashi 15c, Viète 1600, Briggs 1633...

## Isaac Newton 1642-1727

Mathematician and physicist

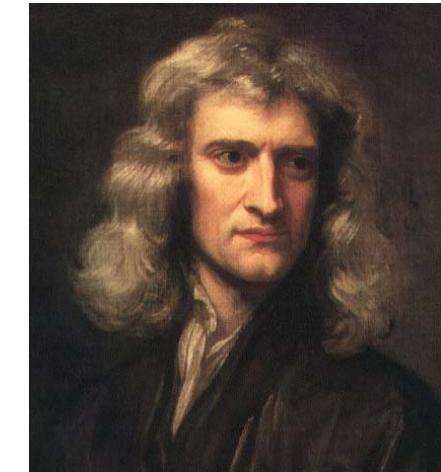
Trinity College, Cambridge, 1661-1696

(BA 1665, Fellow 1667,

Lucasian Professor of Mathematics 1669)

*De analysi per aequationes numero terminorum infinitas* **1669** (published 1711)

After 1696, Master of the Mint



## Joseph Raphson 1648-1715

Mathematician at Jesus College, Cambridge

*Analysis Aequationum universalis* **1690**

Raphson's formulation was better than Newton's ("plus simple" - Lagrange 1798)

FRS 1691, M.A. 1692

Supporter of Newton in the calculus wars—*History of Fluxions*, 1715

## Thomas Simpson 1710-1761

**1740:** *Essays on Several Curious and Useful Subjects...*

1743-1761: Royal Military Academy, Woolwich

Important!—first to treat non-polynomial equations, first to treat systems of eqs.

# Least-squares fitting

## Carl Friedrich Gauss 1777-1855

Mathematics, astronomy, geodesy, magnetism

1792-1795: Braunschweig Collegium Carolinum

**1795**, but not published until 1809

(→ big fight with Legendre)

(During this time as a teenager in Braunschweig he also discovered the binomial theorem, quadratic reciprocity, arithmetic-geometric mean...)

1807-1855: University of Göttingen



## Adrien-Marie Legendre 1752-1833

1791-1833: Académie des Sciences, Paris

**1805** "Sur la méthode des moindres carrés"

applications to orbits of comets



# Gaussian elimination for linear systems of eqs.

**Liu Hui** c. 220 – c. 280

Chinese mathematician  
discusses elimination in his commentaries on

*The Nine Chapters on the Mathematical Art* **263 AD**



**Joseph Lagrange** 1736-1813

Symmetric quadratic forms **1759**



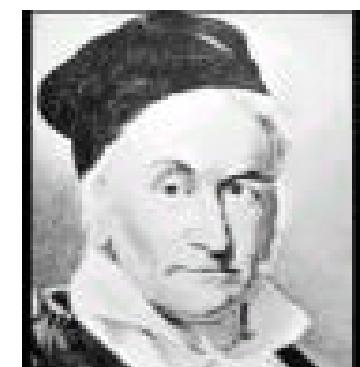
**Carl Friedrich Gauss** 1777-1855

Symmetric systems, normal eqs. **1809**



**Carl Gustaf Jacob Jacobi** 1804-1851

1826-1844: U. of Königsberg  
General systems **1857** (posthumous)



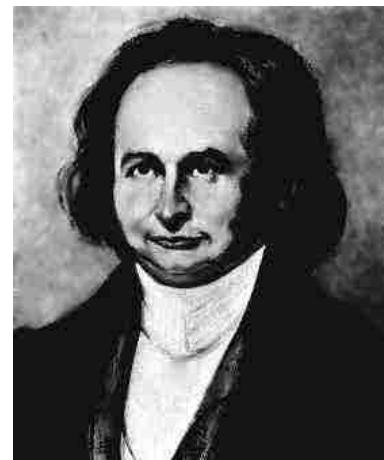
# Gauss quadrature for numerical integration

**Carl Friedrich Gauss** 1777-1855

“Methodus nova integralium valores per approximationem  
inveniendi”, *Comment. Soc. Reg. Sient. Götting. Recent.* **1814**



Gauss did it by continued fractions and hypergeometric functions. Today's more familiar interpretation via orthogonal polynomials was developed by **Jacobi** (1804-1851) in **1826**.



# Adams formulae for ODEs

## Leonhard Euler 1707-1783

1727-1741: St. Petersburg Academy

**1768:** *Institutiones Calculi Integralis*

1741-1766: Berlin Academy

1766-1783: St. Petersburg Academy



## John Couch Adams 1819-1892

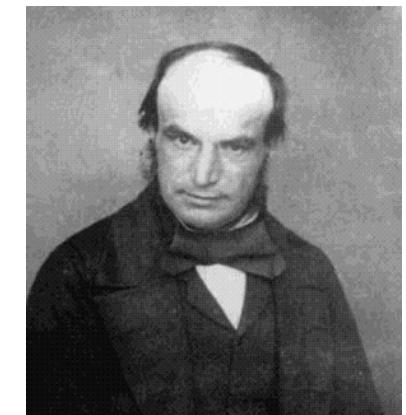
astronomer and mathematician; predicted existence of Neptune

1839-1892: Cambridge U.—Senior Wrangler 1843

**1855?**: work on multistep methods

1858-1892: Lowndean Professor of Astronomy and Geometry

Declined both knighthood and Astronomer Royal post



## Francis Bashforth 1819-1912

influential ballistics expert

1840-1843: Cambridge U.—Second Wrangler 1843

1864-1872: Professor of Applied Mathematics, Royal Military Academy, Woolwich

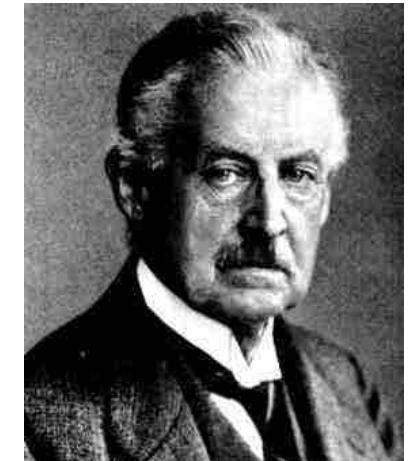
**1883**: paper describing Adams methods (for calculating shapes of drops).

# Runge-Kutta formulae for ODEs

Like Adams formulas, these are a generalization of Euler.  
Coriolis 1830s had some 2<sup>nd</sup>-order formulas. Then —

**Carl David Tolme Runge** 1856-1927

**1895** *Math. Anal.*, “Über die numerische Auflösung...”



**Karl Heun** 1859-1929

PhD. 1881 Göttingen, Prof. Theoretical Mechanics Karlsruhe

**1900** *Zeit. Math. Phys.*, “Neue Methode zur  
approximativen Integration...”

**Martin Wilhelm Kutta** 1867-1944

**1901** general R-K theory, *Zeit. Math. Phys.*,  
“Beitrag zur näherungsweisen Integration...”

Also Nyström 1925, Moulton 1926, von Mises 1930,  
and in the computer era, John Butcher.

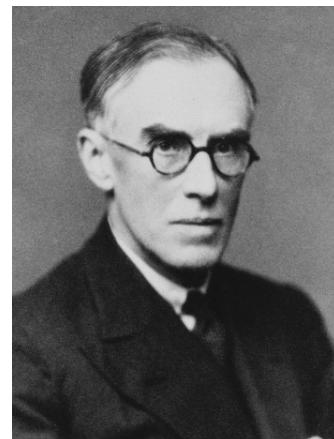


# Finite differences for PDEs

**Lewis Fry Richardson** 1881-1953  
**Richard Southwell** 1888-1970



**Richard Courant** 1888-1972  
**Kurt Friedrichs** 1901-1982  
**Hans Lewy** 1904-1988



**John von Neumann** 1903-1957  
**Peter Lax** ≈1926-



1940 – 1970

# Floating point arithmetic

**Konrad Zuse** 1910-1995

Civil engineer by training

Worked on computers beginning in 1934

“Zuse Apparatebau” company founded in Berlin 1940

Z1 computer, completed in Berlin **1936**

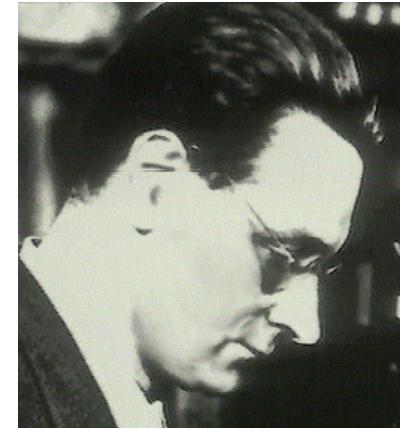
much further developed: Z3 computer, 1941

22-bit floating point binary arithmetic

(14 bits for fraction, 8 for exponent)

1Hz, programmable, stored data but not program

Machine was destroyed in 1945 air raids



Zuse was also an artist.



# Splines

## **Paul de Faget de Casteljau** 1930-

French mathematician/physicist

1958-1992: Citroën; unpublished work in **1958**



## **Pierre Bezier** 1910-1999

1933-1975: engineer at Renault

**1960**: beginning of CADCAM work, Bezier curves

## **Isaac Jacob ("Iso") Schoenberg** 1903-1990

Born in Romania (Landau's son-in-law). To USA in 1930.

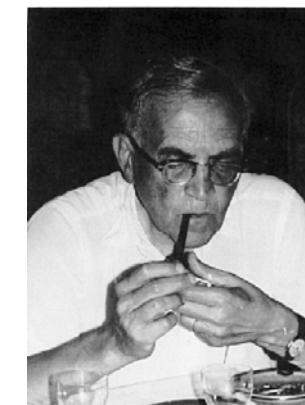
Chicago, Harvard, Princeton, Swarthmore, Colby...

1941-1966: University of Pennsylvania

1943-1945: Army Ballistic Research Laboratory

**1946**: two papers on splines

1966-1973: U. of Wisconsin



## **Carl de Boor** 1937-

Born in what became East Germany. To USA in 1959.

1960-1964: General Motors (grad student intern)

**1962**: first of many publications on splines

Purdue, Michigan...

1972- U. of Wisconsin



# Monte Carlo simulation methods

## **Stanislaw Ulam** 1909-1984

Born in Poland, to USA in 1935, pure mathematician by training  
Princeton, Harvard, Wisconsin, USC

1943-1965: Los Alamos (key figure in hydrogen bomb)

1965-1984: Dept. of Mathematics, U. of Colorado



## **John von Neumann** 1903-1957

Born in Hungary, to USA in 1930, pure mathematician by training  
Manhattan Project, Los Alamos, Atomic Energy Comm.

1930-1957: Princeton U. & Inst. Advanced Study



## **Nicholas Metropolis** 1915-1999

Greek-American, physicist by training

Oscillated between **U. of Chicago** and **Los Alamos**

**1932, 1941, 1945, 1948, 1957, 1965**



**1947:** Invention by Ulam & von N. for applications in neutron diffusion

**1949:** publication of “The Monte-Carlo Method” by Ulam & Metropolis

Also Fermi, Richtmyer, ...

# Simplex algorithm for linear programming

**Leonid Kantorovich** 1912-1986

1934-1960 Professor of Mathematics, Leningrad State U.

**1939:** *Mathematical Methods in the Organization  
and Planning of Production*

1975: Nobel Prize in Economics



**George Dantzig** 1914-2005

1941-1946: Head of Combat Analysis Branch,  
US Air Force Statistical Control

1944: War Department Exceptional Civilian Service Medal

1946: receives PhD at UC Berkeley

**1947:** Simplex algorithm

1948: Koopmans coins expression “linear programming”

1947-1952: Mathematical Advisor, US Defense Department

1952-1960: RAND Corporation

1960-1966: UC Berkeley

1966-2005 : Stanford U.



# Conjugate gradient and Lanczos iterations

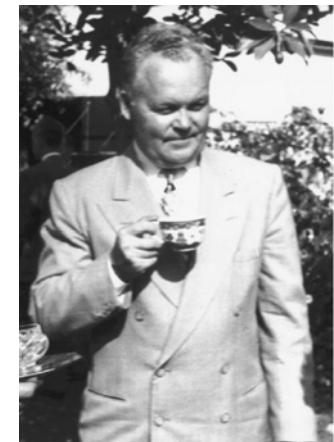
## **Cornelius Lanczos** 1893-1974

Born in Hungary: Fejér, Einstein, ...

1931-1949: Purdue and Boeing

1949-1952: Inst. Numer. Anal., NBS, UCLA

1952-1972: Dublin Inst. Adv. Study, Ireland



## **Magnus Hestenes** 1906-1991

late 1920s-1947: University of Chicago

1947-1973: UCLA

1949-1952: Inst. Numer. Anal., NBS, UCLA



## **Eduard Stiefel** 1909-1978

eminent in geometry and physics as well as computation

Swiss Federal Institute of Technology

**1952:** landmark papers by Lanczos and Hestenes & Stiefel

# Fortran

## John Backus 1924-2007

grew up in Delaware, USA

a poor student; disorganized early career  
with some years in US Army

1949: AB in Mathematics, Columbia U.

1950-1991 IBM

**1954:** first paper about Fortran; programming team is built

1957: Fortran released by IBM

1975: National Medal of Science

1977: Turing Award



# Stiff ODE solvers

**Charles Francis Curtiss** 1921-2007

1938-2007: Dept. Chemistry, U. Wisconsin (student, professor, emeritus) interrupted by govt. work in WWII

**Joseph Oakland Hirschfelder** 1911-1990

1937-1981: Dept. Chemistry, U. Wisconsin

1943-1946: group leader, Los Alamos

1946: Chief Phenomenologist, Bikini Bomb Test

**1952:** “Integration of stiff equations” with Curtiss, *PNAS*

Nat. Academy of Science; Nat. Medal of Science 1976

**Germund Dahlquist** 1925-2005

Royal Institute of Technology, Sweden

**1963:** “A special stability problem for linear multistep methods...”, *BIT*

**C. William Gear** 1935-

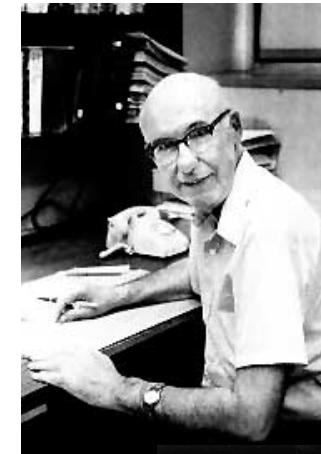
1956-1990: U. of Illinois

1965, 1966 and others: visits to Argonne National Lab

**1967:** first paper on stiff solvers

1971: *Numerical Initial-Value Problems in ODEs*

1990-2000: NEC



# Finite elements for PDE



**Richard Courant** 1888-1972

**1943** "Variational methods..."

(landmark paper, but attracted no notice till later)

Finite elements grew out of the aeronautical engineering of the 1950s.

Additional names include Martin, Turner, Irons, Kelsey, Topp.

**John H. Argyris** 1913-2004

Born in Greece; much of career at U. of Stuttgart, Germany

**1960** *Energy Theorems and Structural Analysis*



**Ray W. Clough** ≈1921-

1950s: Boeing?

**1960** "The finite element in plane stress analysis"

1970- : Professor of Structural Engineering, UC Berkeley  
eminent authority in earthquake engineering

1994: National Medal of Science



Other key early figures include Babushka & Zienkiewicz

# Orthogonal linear algebra

## Wallace Givens 1911-1993

1950s and 1960s: Argonne National Laboratory  
Later, professor at U. of Tennessee  
**1958:** introduction of Givens rotations



## Alston Householder 1904-1993

1946-1969: Oak Ridge National Laboratory  
**1958:** 4-page paper introducing Householder reflections  
1964: *The Theory of Matrices in Numerical Analysis*



## Gene Golub 1932-2007

Professor at Stanford from mid-1960s.  
Key early contributions to many topics including SVD  
and least-squares  
**1965:** “Numerical methods for solving  
linear least-squares problems”



# QR algorithm for matrix eigenvalues

**Heinz Rutishauser** 1918-1970

ETH Zurich

**1958** LR algorithm

**V. N. Kublanovskaya** 1929?-

Steklov Institute of Mathematics, St. Petersburg

**1961** "On some algorithms for the solution of the... eigenvalue problem"

**J. G. F. Francis** 1934-

Late 1950s: National Research Development Corporation, London

Assistant of Christopher Strachey

**1961** "The QR transformation..." I & II, *Computer J.*

**James H. Wilkinson** 1919-1986

Undergraduate in Mathematics at Cambridge

1940-1946: war work related to numerics and ballistics

1946: Turing's assistant on Pilot Ace Computer

1946-1986: National Physical Laboratory

**1965:** *The Algebraic Eigenvalue Problem*

1969: FRS

1970: Turing Award



# Fast Fourier Transform

Gauss 1805 (unpublished) age 28, 2 years before Fourier!

Runge 1903      Yates 1937      Stumpff 1939

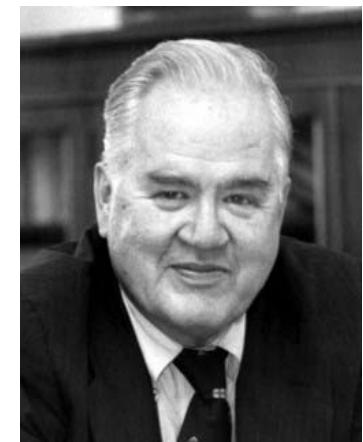
Thomas 1948      Danielson & Lanczos 1942      Good 1958

Wheeler... Gentleman...

Modern birth due to Tukey & Garwin & Sande in 1963, leading to  
**1965** Cooley-Tukey paper in *Mathematics of Computation*

## **John Tukey** 1915-2000

Princeton University, founder of Statistics Dept.  
(also Bell Labs and consultant to U.S., govt. & industry)



## **Richard Garwin** 1928-

Watson Scientific Lab, Columbia U. (later at TJ Watson)  
Well known physicist with major involvement in H-bomb  
FFT motivation related to detection of Soviet nuclear tests.



## **James W. Cooley** 1926-

IBM TJ Watson Research Center. U. Rhode Island.

# Quasi-Newton iterations for optimization

The field was launched between 1959 and 1970.

## William Davidon 1927-

1954 PhD in Physics, U. Chicago

**1959**: “variable metric” report at Argonne National Lab.

(It was finally published in 1991, first issue of *SIOPT*)

1961-1991: Prof. of Physics and Maths, Haverford Coll



## Michael Powell 1936-

1959-1976 Harwell A.E.R.E.

1976- DAMTP, U. of Cambridge

1983 FRS



## Charles Broyden 1933-

1955-1965: English Electric

**1965**: “good” and “bad” Broyden methods

U. College Aberystwyth, U. of Essex

1985?-2003 U. of Bologna



## Roger Fletcher 1939-

1969-1973 Harwell A.E.R.E.... U. of Leeds

**1963**: Davidon-Fletcher-Powell paper

1971-2005 U. of Dundee

2003 FRS

1970 – 2000

# Preconditioning for iterative solution of linear systems

Many people contributed to the discovery of preconditioning, including Evans, Varga, Wachspress, Golub, Concus and O'Leary. Yet there was a particular preconditioner that made the idea famous and is still one of the most effective today: incomplete factorization.

## **Henk van der Vorst** 1944-

1970s-2005: Universities of Delft and Utrecht, Netherlands

**1977**: original paper on incomplete LU factorization

2006: knighted (Ridder in de Orde van de Nederlandse Leeuw)



one of van der Vorst's  
watercolors



# Spectral methods for PDE

Important work in 1950s and 1960s by Lanczos,  
Clenshaw, Elliott, Fox and Mason et al.

Contributions also from Kreiss and Oliger and others.  
These methods were made famous by:

## **Steve Orszag** 1943-

1966?-1984: Applied Mathematics, MIT

**1971**: series of major papers on spectral methods in fluid mechanics  
(Orszag coined the phrase “spectral methods”)

1984-1998: Prof. of Applied Mathematics, Princeton U.

1998- Prof. of Mathematics, Yale U.

Orszag's son Peter is currently Director of Office of Management & Budget



## **David Gottlieb** 1944-2008

From Israel; came to USA in 1972

1972-1976: MIT and ICASE (NASA Langley)

**1977**: spectral methods book by D.G. and S.A.O.

1976-1985: Dept. of Applied Mathematics, Tel-Aviv U.

1985-2008: Prof. of Applied Mathematics, Brown U.



# MATLAB

**Cleve Moler** 1939-

Author of EISPACK, LINPACK, four textbooks

high school Utah, BA Caltech, PhD Stanford

1965-1973: U. of Michigan

1973-1984: U. of New Mexico

strong links with Argonne National Laboratory

**1977**: creation of first version of Matlab

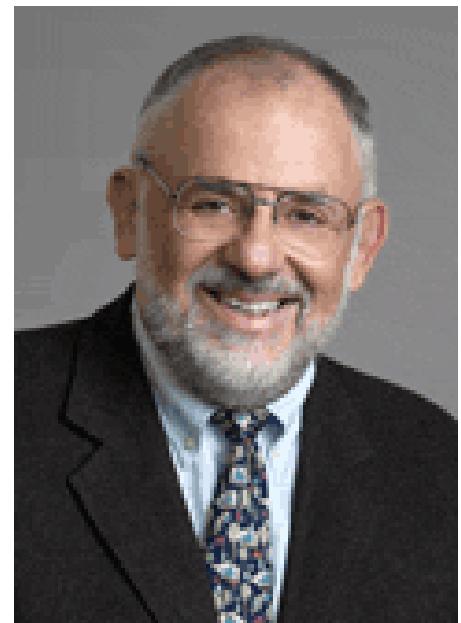
1984: Jack Little founds MathWorks

1985: first Matlab sale

1984-1989: Moler employed at Intel and Ardent

1989: joins MathWorks as Chief Scientist

N.B.: Matlab is 60% as old as Fortran!



# Multigrid Methods for PDE

**R. P. Fedorenko** 1930-

1961: invention of 2-grid and later multigrid method.

This work extended also by N. S. Bakhvalov, 1966.

**Achi Brandt** 1938-

1963- : Applied Mathematics, Weizmann Institute, Israel

**1973**: first paper on multigrid methods

1977: 57-page paper in *Mathematics of Computation*



**Wolfgang Hackbusch** 1948-

**1976**: Independent rediscovery of multigrid

1982- : Professor of Applied Maths., U. Kiel

199?-: director of Max-Planck Inst. In Leipzig



# IEEE arithmetic

**William (“Velvel”) Kahan** 1933-

late 1960s- : Dept. of Mathematics, UC Berkeley

**1977**: the draft IEEE floating-point standard released

1985: adoption of the standard after much wrangling

1989: Turing Award



# Nonsymmetric Krylov iterations for large matrix problems

Many contributors including Arnoldi, Elman, Schultz, Freund, Gutknecht.

**P. K. W. Vinsome** 19??-

Shell Petroleum Co.

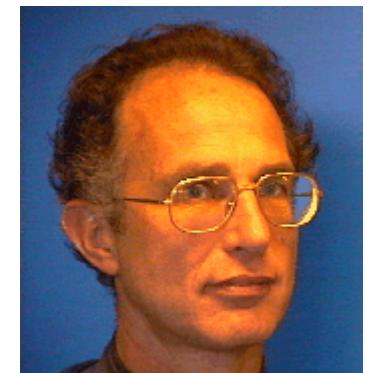
**1976** paper on Orthomin



Youcef **Saad** 1950-

Yale University, U. of Minnesota

**1986**: GMRES paper with Schultz

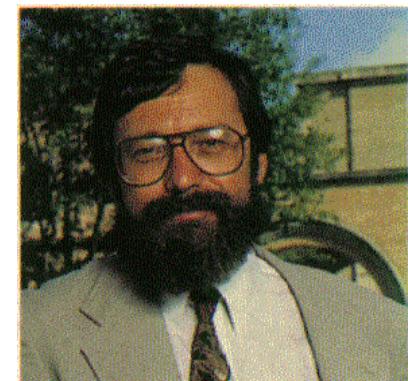


**Henk van der Vorst** 1944-

Professor of Mathematics at U. of Utrecht

**1986**: BiCGSTAB paper—most cited maths paper in 1990s

**1996**: Jacobi-Davidson paper with Sleijpen



**Dan Sorensen** 1947-

Argonne National Laboratory

Rice U.

**1992**: implicitly restarted Arnoldi

**1996**: *ARPACK User's Guide* with Lehoucq and Yang

# Interior Point Methods for optimization

Earlier work by Carroll (1961) and Khachiyan (1979) and also by Fiacco & McCormick (1968), Margaret Wright (1976) and others on barrier methods.

## Narendra Karmarkar 1957-

1978: BTech in Elect. Engr., IIT Bombay

1982?: PhD, U. C. Berkeley

1983-? AT&T Bell Labs

**1984** :“A new polynomial time algorithm for linear programming,” *Combinatorica*

Now lives in India



# Fast Multipole Method for $N$ -body simulation and more

Related earlier work by Barnes & Hut & others

## Vladimir Rokhlin 1952–

Born in USSR; to USA in late 1970s

1976-1985: Exxon Production Research Co.

1983: PhD in Applied Mathematics, Rice U.

**1985** “Rapid solution of integral equations...”

1985- Prof. of Computer Science, Yale U.



## Leslie Greengard 1958–

From Boston, New York, New Haven

1987 M.D. and Ph.D. (Comp. Sci.) Yale U.

**1987** “A fast algorithm for particle simulations”, with Rokhlin

1989- Prof. of Mathematics, Courant Inst., NYU

Currently serving as Director.



Both VR and LG have eminent fathers.

# Wavelets

**Jean Morlet** 1931-2007

Geophysicist at Elf Aquitaine / Oric

Work beginning 1975 leads to major publication **1982**



Also Alex Grossmann 1984, Stephane Mallat 1989,  
Yves Meyer 1986

**Ingrid Daubechies** 1954-

Training in physics and mathematics

From Belgium; came to USA in 1987

1975-1987: Vrije Universiteit Brussel

1987-1994: AT&T Bell Labs; Rutgers U.

Big change in 1980s physics → mathematics

**1988**: “Orthonormal bases of compactly supported wavelets”

1993- : Princeton U.

MacArthur Prize and many others



# Automatic differentiation

Many antecedents including Beda (1959), Wengert (1964), Speelpenning (1980), Kedem (1980), Rall (1981), Baur and Strassen (1984)... more recently Bischof & Carle (ADIFOR, 1991) and many others.

A central figure in the modern rebirth of these ideas (in particular the use of “reverse mode”) has been

**Andreas Griewank** 1950-

Argonne National Laboratory  
Institute for Scientific Computing, TU Dresden  
Humboldt-University Berlin



# *The Inventors*

Adams	Argyris	Backus	Bashforth
Bezier	Brandt	Broyden	Clough
Cooley	Courant	Curtiss	Dahlquist
Dantzig	Daubechies	Davidon	de Boor
de Casteljau	Euler	Fedorenko	Fletcher
Francis	Friedrichs	Garwin	Gauss
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	Hirschfelder	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	Metropolis
Moler	Morlet	von Neumann	Newton
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	Southwell	Stiefel
Tukey	Ulam	van der Vorst	Vinsome
Wilkinson	Zuse		

# Who was an engineer?

Adams	<b>Argyris</b>	<b>Backus</b>	Bashforth
<b>Bezier</b>	Brandt	Broyden	<b>Clough</b>
<b>Cooley</b>	Courant	Curtiss	Dahlquist
Dantzig	Daubechies	Davidon	de Boor
de Casteljau	Euler	Fedorenko	Fletcher
Francis	Friedrichs	Garwin	Gauss
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	Hirschfelder	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	Metropolis
Moler	Morlet	von Neumann	Newton
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	<b>Southwell</b>	Stiefel
Tukey	Ulam	van der Vorst	Vinsome
Wilkinson	<b>Zuse</b>		

# *Who was a physicist?*

Adams	Argyris	Backus	Bashforth
Bezier	Brandt	Broyden	Clough
Cooley	Courant	Curtiss	Dahlquist
Dantzig	<b>Daubechies (½)</b>	<b>Davidon</b>	de Boor
<b>de Casteljau</b>	Euler	Fedorenko	Fletcher
Francis	Friedrichs	<b>Garwin</b>	<b>Gauss (½)</b>
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	Hirschfelder	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	<b>Metropolis</b>
Moler	<b>Morlet</b>	von Neumann	<b>Newton (½)</b>
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	Southwell	Stiefel
Tukey	Ulam	van der Vorst	<b>Vinsome</b>
Wilkinson	Zuse		

# *Who was a chemist?*

Adams	Argyris	Backus	Bashforth
Bezier	Brandt	Broyden	Clough
Cooley	Courant	<b>Curtiss</b>	Dahlquist
Dantzig	Daubechies	Davidon	de Boor
de Casteljau	Euler	Fedorenko	Fletcher
Francis	Friedrichs	Garwin	Gauss
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	<b>Hirschfelder</b>	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	Metropolis
Moler	Morlet	von Neumann	Newton
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	Southwell	Stiefel
Tukey	Ulam	van der Vorst	Vinsome
Wilkinson	Zuse		

# *Who was a mathematician?*

Adams	Argyris	Backus	Bashforth
Bezier	Brandt	Broyden	Clough
Cooley	Courant	Curtiss	Dahlquist
Dantzig	Daubechies (½)	Davidon	de Boor
de Casteljau	Euler	Fedorenko	Fletcher
Francis	Friedrichs	Garwin	Gauss (½)
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	Hirschfelder	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	Metropolis
Moler	Morlet	von Neumann	Newton (½)
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	Southwell	Stiefel
Tukey	Ulam	van der Vorst	Vinsome
Wilkinson	Zuse		

(Including computer scientists and statisticians, since very hard to distinguish)

# *Who was a professor?*

Adams	Argyris	Backus	Bashforth
Bezier	Brandt	Broyden	Clough
Cooley	Courant	Curtiss	Dahlquist
<b>Dantzig</b>	<b>Daubechies</b>	<b>Davidon</b>	<b>de Boor</b>
de Casteljau	Euler	Fedorenko	Fletcher
Francis	Friedrichs	Garwin	Gauss
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	Hirschfelder	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	Metropolis
Moler	Morlet	von Neumann	Newton
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	Southwell	Stiefel
Tukey	Ulam	van der Vorst	Vinsome
Wilkinson	Zuse		

(Including English academics like Raphson with titles other than professor)

# *Who had major involvement with government or industry?*

Adams	<b>Argyris</b>	<b>Backus</b>	<b>Bashforth</b>
<b>Bezier</b>	Brandt	<b>Broyden</b>	<b>Clough</b>
<b>Cooley</b>	<b>Courant</b>	Curtiss	Dahlquist
<b>Dantzig</b>	<b>Daubechies</b>	<b>Davidon</b>	de Boor
<b>de Casteljau</b>	Euler	Fedorenko	<b>Fletcher</b>
<b>Francis</b>	Friedrichs	<b>Garwin</b>	<b>Gauss</b>
<b>Gear</b>	<b>Givens</b>	Golub	Gottlieb
Greengard	<b>Griewank</b>	Hackbusch	<b>Hestenes</b>
Heun	<b>Hirschfelder</b>	<b>Householder</b>	Liu
Jacobi	<b>Kahan</b>	<b>Kantorovich</b>	<b>Karmarkar</b>
Kublanovskaya	Kutta	Lagrange	<b>Lanczos</b>
Lax	Legendre	Lewy	<b>Metropolis</b>
<b>Moler</b>	<b>Morlet</b>	<b>von Neumann</b>	Newton
Orszag	<b>Powell</b>	Raphson	Richardson
<b>Rokhlin</b>	Runge	Rutishauser	Saad
<b>Schoenberg</b>	<b>Sorensen</b>	<b>Southwell</b>	Stiefel
Tukey	<b>Ulam</b>	van der Vorst	<b>Vinsome</b>
<b>Wilkinson</b>	<b>Zuse</b>		

59%

(i.e., near the time of their big contributions)

# *Who was born in the USA?*

Adams	Argyris	<b>Backus</b>	Bashforth
Bezier	Brandt	Broyden	<b>Clough</b>
<b>Cooley</b>	Courant	<b>Curtiss</b>	Dahlquist
<b>Dantzig</b>	Daubechies	<b>Davidon</b>	de Boor
Bezier	Euler	Fedorenko	Fletcher
Francis	Friedrichs	<b>Garwin</b>	Gauss
Gear	<b>Givens</b>	<b>Golub</b>	Gottlieb
<b>Greengard</b>	Griewank	Hackbusch	<b>Hestenes</b>
Heun	<b>Hirschfelder</b>	<b>Householder</b>	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	<b>Metropolis</b>
<b>Moler</b>	Morlet	von Neumann	Newton
<b>Orszag</b>	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	<b>Sorensen</b>	Southwell	Stiefel
<b>Tukey</b>	Ulam	van der Vorst	Vinsome
Wilkinson	Zuse		

# *Who ended up in the USA?*

Adams	Argyris	<b>Backus</b>	Bashforth
Bezier	Brandt	Broyden	<b>Clough</b>
<b>Cooley</b>	<b>Courant</b>	<b>Curtiss</b>	Dahlquist
<b>Dantzig</b>	<b>Daubechies</b>	<b>Davidon</b>	<b>de Boor</b>
de Casteljau	Euler	Fedorenko	Fletcher
Francis	<b>Friedrichs</b>	<b>Garwin</b>	Gauss
<b>Gear</b>	<b>Givens</b>	<b>Golub</b>	<b>Gottlieb</b>
<b>Greengard</b>	Griewank	Hackbusch	<b>Hestenes</b>
Heun	<b>Hirschfelder</b>	<b>Householder</b>	Liu
Jacobi	<b>Kahan</b>	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
<b>Lax</b>	Legendre	<b>Lewy</b>	<b>Metropolis</b>
<b>Moler</b>	Morlet	<b>von Neumann</b>	Newton
<b>Orszag</b>	Powell	Raphson	Richardson
<b>Rokhlin</b>	Runge	Rutishauser	<b>Saad</b>
<b>Schoenberg</b>	<b>Sorensen</b>	Southwell	Stiefel
<b>Tukey</b>	<b>Ulam</b>	van der Vorst	Vinsome
Wilkinson	Zuse		

# How old were they?

eligible for the Fox Prize!

Adams 36	Argyris 47	Backus 30	Bashforth 64
Bezier 50	Brandt 35	Broyden 32	Clough 39
Cooley 39	Courant 40,57	Curtiss 31	Dahlquist 38
Dantzig 33	Daubechies 34	Davidon 32	de Boor 25
de Casteljau 28	Euler 59	Fedorenko ?	Fletcher 24
Francis 27	Friedrichs 27	Garwin 37	Gauss 18,32,37
Gear 32	Givens 47	Golub 33	Gottlieb 33
Greengard 29	Griewank 40	Hackbusch 28	Hestenes 46
Heun 41	Hirschfelder 41	Householder 54	Liu 43
Jacobi 22,40	Kahan 44	Kantorovich 27	Karmarkar 27
Kublanovskaya 32	Kutta 34	Lagrange 23	Lanczos 59
Lax 35	Legendre 53	Lewy 24	Metropolis 33
Moler 38	Morlet ?	von Neumann 44	Newton 27
Orszag 28	Powell 27	Raphson 42	Richardson 35
Rokhlin 33	Runge 45	Rutishauser 28	Saad 36
Schoenberg 43	Sorensen 45	Southwell 52	Stiefel 43
Tukey 50	Ulam 38	van der Vorst 33,42	Vinsome ?
Wilkinson 46	Zuse 26		

So, ivory tower or coal face?  
The answer seems to be a blend:

Most of the big algorithms were invented by  
**academic mathematicians**  
who had  
**MAJOR involvement**  
with applications in industry or government.

*What is the first great numerical algorithm of the 21<sup>st</sup> century?*

