Homework 18

1. (Another method for eliminating the secular term.) Consider Duffing's equation
$$y'' + y + \varepsilon y^3 = 0, \quad y(0) = 1, y'(0) = 0.$$
In the method of averaging, we consider the integral $I = \int_0^{2\pi} Y_0(\partial^2 Y_1/\partial t^2 + Y_1)dt$, taken over the short time scale period of oscillation. Y_0 and Y_1 are defined as
$$y(t) \sim Y_0(t, \tau) + \varepsilon Y_1(t, \tau) + \cdots, \quad \tau = \varepsilon t. \tag{1}$$
Throughout this integration the long time scale τ remains fixed, and thus Y_0 and Y_1 should be periodic in t.
   (a) Show that if Y_0 and Y_1 are periodic in t and (1) is uniformly valid, then $I = 0$.
   (b) Use
   $$\frac{\partial^2 Y_1}{\partial t^2} + Y_1 = -Y_0^3 - 2\frac{\partial^2 Y_0}{\partial t \partial \tau}$$
   and the requirement that $I = 0$ to derive equation
   $$-3A^2 A^* - 2i dA/d\tau = 0.$$
2. Use multiple scale analysis to find a leading-order approximation to
$$y'' + y + \varepsilon y' y^2 = 0, \quad y(0) = 1, y'(0) = 0, \varepsilon > 0.$$