Homework 16

1. Show that, for the Schrödinger equation $\epsilon^2 y'' = Q(x)y$, if $S_2' = 0$ then $S_n' = 0$ for $n \geq 2$. Deduce that the most general function $Q(x)$ for which the equation $\epsilon^2 y'' = Q(x)y$ has the physical-optics approximation as its exact solution is $Q(x) = (c_1 x + c_2)^{-4}$.

2. (Optional) Use WKB theory to obtain the solution to

$$\epsilon y'' + a(x)y' + b(x)y = 0[a(x) > 0, y(0) = A, y(1) = B]$$

correct to order ϵ.

3. Consider the equations

$$\epsilon^2 y''(x) = (\sin x)y, \epsilon^2 y''(x) = (\sin x^2)y, \epsilon^2 y''(x) = [1 + (\sin x)^2]y.$$

For which fixed values of x is the WKB physical-optics approximation a good approximation to $y(x)$ as $\epsilon \to 0$. Is WKB accurate as $x \to \infty$.