Homework 15

1. Derive the inner solution for the boundary layer of the ODE

$$\varepsilon y'''(x) - y'(x) + xy(x) = 0, \quad y(0) = y'(0) = y(1) = 1$$

at $x = 1$ up to order ε. That is, derive the expansion

$$Y_{in}(Z) = Y_0(Z) + \varepsilon^{\frac{1}{2}} Y_{1/2}(Z) + \varepsilon Y_1(Z) + \cdots$$

where $Z = (1-x)/\delta$, where δ is the boundary layer thickness.

2. Assume that $a(x) \sim \alpha x$, $b(x) \sim \beta$ as $x \to 0+$ and $\alpha, \beta > 0$, prove that

$$\int_x^1 \frac{b(t)}{a(t)} dt \sim -\frac{\beta}{\alpha} \ln x \quad x \to 0+$$

and

$$\int_x^1 \frac{b(t)}{a(t)} dt + \frac{\beta}{\alpha} \ln x \sim \int_0^1 \left(\frac{b(t)}{a(t)} - \frac{\beta}{\alpha t} \right) dt \quad x \to 0+$$

under suitable regularity condition on $a(t)$ and $b(t)$.