Homework 13

1. Prove the estimates

$$\int_\delta^\infty e^{-t-\varepsilon/t}dt = \int_\delta^\infty e^{-t}(1-\varepsilon/t)dt + O(\varepsilon^2/\delta), \quad \varepsilon/\delta \to 0$$

and

$$\int_\delta^\infty e^{-t-\varepsilon/t}dt = \int_\delta^\infty e^{-t}\left(1-\frac{\varepsilon}{t}+\frac{\varepsilon^2}{2t^2}-\frac{\varepsilon^3}{6t^3}+\frac{\varepsilon^4}{24t^4}-\frac{\varepsilon^5}{120t^5}\right)dt+O\left(\frac{\varepsilon^6}{\delta^5}\right), \quad \varepsilon/\delta \to 0$$

by showing that

$$\left|e^{-x}-\sum_{n=0}^{N}\frac{(-x)^n}{n!}\right| \le \frac{x^{N+1}}{(N+1)!}$$

for all $x \ge 0$.