

Homework 10

1. Perform a local analysis of the *algebraic* equation $y = e^{xy}$ near $x = 1/e$ by substituting $y = e + \delta(x)$, where $\delta \rightarrow 0$ as $x \rightarrow 1/e$. Solve approximately for $\delta(x)$ to show that near $x = 1/e$, $y(x)$ has a square-root singularity.
2. Does the solution to the initial-value problem $y'(x) = \sqrt{x^2 + y^2}$ [$y(0) = a$] remain finite for all x ?
3. Show that the leading behavior of an explosive singularity of the Thomas-Fermi equation $y'' = y^{3/2}x^{-1/2}$ is correctly given by

$$y(x) \sim \frac{400a}{(x-a)^4}, \quad x \rightarrow a.$$

4. Let $yd^4y/dx^4 = 1$ [$y(0) = y''(0) = y(1) = y''(1) = 0$]. Find the asymptotic behavior of $y(x)$ as $x \rightarrow 0+$. Try several terms involving combinations of logs and powers.
5. Find the leading asymptotic behavior of the solution to $yy'' = x^3y'^2$ as $x \rightarrow +\infty$.