

Homework 06

1. Classify the points at 0 and ∞ of the following differential equations:

(a). $x^7 d^4 y/dx^4 = y'$;

(b). $x^3 y''' = y$;

(c). $y''' = x^3 y$;

(d). $x^2 y'' = e^{1/x} y$;

(e). $(\tan x)y' = y$;

(f). $y'' = (\ln x)y$.

2. Find the Taylor series about 0 of the solution to the initial-value problems:

(a). $y'' - 2xy' + 8y = 0$ $[y(0) = 0, y'(0) = 4]$;

(b). $y'' = (x - 1)y$ $[y(0) = 1, y'(0) = 0]$.

3. The leading behavior of a particular solution to $x^3 y'' = y$ is $y(x) \sim x(x \rightarrow \infty)$. What is the next largest term in the expansion of $y(x)$ for large positive x .

4. Find series expansions of all the solutions to the following differential equations about $x = 0$. Try to sum in closed form any infinite series that appear.

(a). $y'' + (e^x - 1)y = 0$;

(b). $2xy'' - y' + x^2 y = 0$;

(c). $xy'' + (\frac{1}{2} - x)y' - y = 0$.

5. Perform a local analysis of solutions to $(x - 1)y'' - xy' + y = 0$ at $x = 1$. Use the results of this analysis to prove that a Taylor series expansion of any solution about $x = 0$ has an infinite radius of convergence.