

Homework 05

1. Prove that the identity

$$[\hat{N}, a^\dagger] = a^\dagger,$$

where $[A, B] = AB - BA$ is the commutator, $\hat{N} = a^\dagger a$.

2. Prove that

$$|n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle, \quad n = 0, 1, \dots$$

satisfies $\langle n|n'\rangle = \delta_{nn'}$ by induction, where we have already taken $\langle 0|0\rangle = 1$.

3. Prove that $He_n(x) = 2^{-n/2} H_n(x/\sqrt{2})$, where

$$H_n(x) = e^{\frac{x^2}{2}} \left(x - \frac{d}{dx} \right)^n e^{-\frac{x^2}{2}}$$

$$He_n(x) = (-1)^n e^{\frac{x^2}{2}} \left(\frac{d}{dx} \right)^n e^{-\frac{x^2}{2}}$$