Homework 05

1. Suppose $y_1(x)$ is a particular solution of the ODE

$$Ly = y^{(n)} + p_{n-1}(x)y^{(n-1)} + \cdots + p_0(x)y = 0.$$

   Define $y(x) = u(x)y_1(x)$. Prove that $v = u'(x)$ satisfies a linear homogenous ODE of order-$(n-1)$.

2. Formulate the method of variation of parameters for a third-order linear equation. How does it work for an nth-order equation?

3. Show that a necessary and sufficient condition that

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

   be exact is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

4. Show that any equation of the following form

$$a(x)y''(x) + b(x)y'(x) + c(x)y(x) + d(x)Ey(x) = 0, \quad y(\alpha) = y(\beta) = 0$$

   can be transformed to the following *Sturm-Liouville* form

$$\frac{d}{dx}[p(x)\frac{dy}{dx}] + [q(x) + Er(x)]y = 0, \quad y(\alpha) = y(\beta) = 0. \tag{1}$$