Homework 09

1. 对 $A_h = \frac{1}{h^2}\begin{pmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ & \ddots & \ddots & -1 \\ & & -1 & 2 \end{pmatrix}$ 采用Gauss_Seidel方法,

证明

$$\mu_k^{GS} = \cos^2 \frac{k\pi}{n}$$

$$W_k^{GS}(j) = \left(\cos\frac{k\pi}{n}\right)^j \sin\frac{jk\pi}{n}$$

2. 设 $W_k^h(j) = \sin(jk\pi h)$ $\quad h = 1/n$

$$I_h^{2h} = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \cdots & 0 & 0 \\ \cdots & & & & \cdots & & \cdots & \\ 0 & 0 & 0 & 0 & 0 & \cdots & \frac{1}{2} & \frac{1}{4} \end{pmatrix}, \quad I_{2h}^h \triangleq 2\left(I_h^{2h}\right)^T$$

证明:

① $I_h^{2h} W_k^h = C_k W_k^{2h}$ $\qquad C_k = \cos^2(k\pi/2n)$

$I_h^{2h} W_{k'}^h = -S_k W_k^{2h}$ $\qquad S_k = \sin^2(k\pi/2n)$

$k' = n-k$, $1 \le k < n/2$, $\quad I_h^{2h} W_{n/2}^h = 0$

② $I_{2h}^h W_k^{2h} = C_k W_k^h - S_k W_{k'}^h$, $\quad 1 \le k < n/2$

$k' = n-k$.