

Homework 06

便笺标题

2011/3/27

1. 定义 n 次 Bernoulli 多项式 $B_n(x)$

$$\begin{cases} B_0(x) = 1 & x \in [0, 1] \\ B'_n(x) = B_{n-1}(x) & \text{且} \int_0^1 B_n(x) dx = 0, \quad n=1, 2, \dots \end{cases}$$

由此得

$$B_n(0) = B_n(1), \quad n=2, 3, \dots$$

及

$$B_n(x) = (-1)^n B_n(1-x), \quad n=0, 1, 2, \dots$$

定义 $\tilde{B}_n(x)$ 为 B_n 的周期扩张, 即

$$\begin{aligned} \tilde{B}_n(x) &= B_n(x), \quad x \in [0, 1] \\ \tilde{B}_n(x+1) &= \tilde{B}_n(x), \quad x \in \mathbb{R} \end{aligned}$$

证明 $\tilde{B}_n$ 有 Fourier 级数展开式:

$$\tilde{B}_{2m}(x) = 2(-1)^{m-1} \sum_{k=1}^{+\infty} \frac{\cos(2k\pi x)}{(2k\pi)^{2m}}, \quad m=1, 2, \dots$$

$$\tilde{B}_{2m+1}(x) = 2(-1)^m \sum_{k=1}^{+\infty} \frac{\sin(2k\pi x)}{(2k\pi)^{2m+1}}, \quad m=1, 2, \dots$$