

1. 设互异节点  $x_0, x_1, \dots, x_n$ ,  $l_k(x)$  为 Lagrange 插值基函数, 证明.

$$(1) \sum_k l_k(x) = 1$$

$$(2) \sum_k x_k^j l_k(x) = x^j \quad j=0, 1, \dots, n$$

$$(3) \sum_k (x_k - x)^j l_k(x) = 0 \quad j=1, \dots, n$$

2. 证明 Newton 差商的各条性质.

3. 对 Lebesgue 常数证明

$$\lambda_n(x; X) = \sum_{k=0}^n |l_k(x)|$$

$$\Delta_n(X) = \|\lambda_n(x; X)\|_\infty.$$