1. Let $Z \sim N(0, \sigma^2)$. Show that

$$\sup_{t>0} \left\{ P(Z \geq t) e^{t^2/(2\sigma^2)} \right\} = \frac{1}{2}.$$

2. Consider the covariance matrix $\boldsymbol{\Sigma} = (\sigma_{ij})$ with an autoregressive Toeplitz structure: $\sigma_{ij} = \rho^{|i-j|}$ with $0 < |\rho| < 1$. Show that the irrepresentable condition

$$\| \boldsymbol{\Sigma}_{S^c S} (\boldsymbol{\Sigma}_{SS})^{-1} \operatorname{sgn}(\boldsymbol{\beta}_S^*) \|_\infty \leq \alpha < 1$$

holds and identify the constant α.

3. Derive the ADMM algorithm for the group Lasso problem

$$\min_{\boldsymbol{\beta}} \left\{ \frac{1}{2} \| \mathbf{y} - \mathbf{X}\boldsymbol{\beta} \|_2^2 + \lambda \sum_{g=1}^{G} \| \boldsymbol{\beta}_g \|_2 \right\},$$

where $\boldsymbol{\beta} = (\boldsymbol{\beta}_1^T, \ldots, \boldsymbol{\beta}_G^T)^T$.

4. ESL Ex. 3.30

5. ESL Ex. 4.2

6. ESL Ex. 4.3

7. ESL Ex. 4.5

8. ESL Ex. 4.7