Image Reconstruction

Course Introduction

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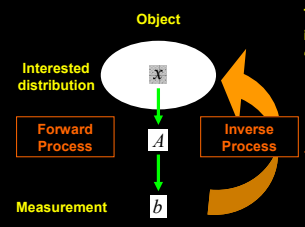
Outline

- Image reconstruction problem
- Required preliminaries
- Syllabus
- References and resources
- Examination scheme

Image reconstruction problem
an inverse problem



The problem is to reconstruct the image from measured data by "solving" the following equation

$$A(x) = b$$

- **Discrete formulation: systems of linear equations with discrete configuration in finite dimensional spaces**
- **Continuous formulation: operator equation with continuous configuration in function spaces**

In most cases, A is linear.

Reconstruction approaches

- In some problems, there are exact analytic inverse formulas.
- In some problems, there are not analytic inverse formulas. Iterative methods based on statistical criteria are utilized.
- Approximate inverse are advocated in practice.
  - In X-ray CT, the inverse formula by Radon is not preferred, but the approximate filtered backprojection (FBP) is utilized.

Ill-posed problems
characteristics of inverse problems

- Existence
  - not an issue in practical applications.
- Uniqueness may fail, due to
  - insufficient data
  - the nature of the physical process
  - e.g., inverse source in bioluminescence tomography

To be addressed in this course.

- Stability could fail
  - small perturbation in measurement may lead to big difference in solutions.
  - typical with image reconstruction problem.
    - limited bandwidth in practice
    - noisy/inconsistent/incomplete data
  - e.g., inverse heat conduction.

Bertero, M. and P. Boccacci (1998). pp. 3 – 5.

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Required preliminaries

- 实分析，泛函分析，偏微分方程
- 数值计算
- 信号处理，图像处理
- 统计学
- X射线物理，光学
- ....

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Syllabus 1

- **1 Preliminaries**
  - Hibert spaces, Banach spaces
  - Function Spaces, *L*1, *L*2, *Lp*, C^m, C^\infty,
  - Local convex TVS, Schwartz space, Distributions
  - Fourier theory for *L*1, *L*2 and Distributions
  - Special functions: Bessel functions, Gegenbauer Polynomials, Spherical Harmonics
  - Integral Operators: Hilbert Transform, Singular Integral Operators with Odd Kernels, Riesz Transforms, Riesz Potentials, Mellin Transform, Hankel Transform
  - Sobolev spaces
- **2 Integral Geometry**
  - Radon Transform, Inversion via Riesz Potentials, via Spherical Harmonics
  - Ray Transform, Inversion via Riesz Potentials
  - Cone Beam Transform, Inversion Formulas

Syllabus 2

- **3 Sampling Theory**
  - Paley-Wiener Theory
  - Poisson's Summation Formula
  - Shannon's Sampling Theorem
  - Sampling on Oblique Grids
- **4 Regularization and SVD**
  - Moore-Penrose inverse
  - Tikhonov's Theory for Ill-posed Problem
  - Singular Value Decomposition
  - Regularization

Syllabus 3

- **5 Statistical Inference**
  - Bayesian Inference
  - Prior Information
  - Data Models, Gaussian Noise, Poissonian noise, Poissonian + Gaussian Mixed Noise.
- **6 Markov Random Fields**
  - Markov Random Fields
  - Gibbsian Random Fields
  - Gibbs sampler
  - Geman's Discrete Model
  - Mumford and Shah's Model
  - Algorithms

Syllabus 4

- **7 Optmization Tools**
  - Gradient descent
  - Incremental Gradient descent
- **8 Iterative Image Reconstruction Methods**
  - EM algorithm
  - Landweber Scheme
  - Projection methods，Bregman and f- divergences
  - Superorization methods,
- **9 Approximate Inverse**
  - Approximate Inverse
  - reconstruction of features

Syllabus 5

- **10 Generations of CT**
  - Parallel Scanning Geometry, Fan Beam Scanning Geometry, 3D Helical/Spiral Fan Beam Scanning, 3D Cone Beam Scanning
- **11 Reconstruction Methods of CT**
  - Parallel Scanning in 2D
  - Fan Beam Scanning in 2D
  - Cone beam scanning in 3D
- **12 SVD of Radon Transform**
  - SVD of Radon Transform
  - Null space of Radon transform
- **13 Sampling Theory & Resolution of CT**
  - Semi-continuous Case
  - Discrete Case
- **14 Iterative methods**
  - Application of previous iterative methods
  - Approximate Inverse/reconstruction of features,
  - snark09 software
  - Evaluation of reconstruction methods

Syllabus 6

- **16 Radiative Transfer Equation**
  - The Radiation Field
  - Interaction of the Radiation Field with Matter
  - The Equation of Radiative Transfer
  - Boundary Conditions on the Equation of Transfer
  - Diffusion Approximation of RTE
- **18 Optical Molecular imaging Modalities**
  - DOT, FMT, BLT
- **19 Bioluminescence Tomography**
  - BLT based on RTE
  - BLT Based on Diffusion Approximation
  - Solution Structure of BLT
  - Reconstruction Methods for BLT
  - Multi-spectral Bioluminescence Tomography

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References

- Censor, Y. and S. A. Zenios (1997). Parallel optimization : theory, algorithms, and applications. New York, Oxford University Press.
- Louis, A. K. (1989). Inverse und schlecht gestellte Probleme. Stuttgart, B.G. Teubner
- Natterer, F. (2001). The mathematics of computerized tomography. Philadelphia, Society for Industrial and Applied Mathematics.
- Natterer, F. and F. Wübbeling (2001). Mathematical methods in image reconstruction. Philadelphia, Society for Industrial and Applied Mathematics.
- Stein, E. M. and G. L. Weiss (1975). Introduction to Fourier analysis on Euclidean spaces. Princeton, N.J., Princeton University Press.
- Gilbarg, D. and N. S. Trudinger (2001). Elliptic partial differential equations of second order. Berlin ; New York, Springer.

Resources

- MathNet
- IEEE Xplore
- EBSCO
- ISI Web of Science
- Google at hk

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Examination

- Submit a written report for a topic in this course.
- Discuss it with me before starting up.
- Implement at least two methods, conduct sufficient numerical simulation, and compare their performance.
- The structure of the report is as follows
  - A introduction
  - B fundamentals
  - C methods
  - D materials
  - E experiments
  - F results
  - G discussions
  - H conclusions
- **No Plagiarism.**