Density of hyperbolicity and homoclinic bifurcations. Classes $4, 5, 6$.

S. Crovisier - E. R. Pujals

August 2009

Partial hyperbolic case: $E^s \oplus E^c \oplus E^u$

Generically, given $f \in Diff^1(M^n)$ and H_p

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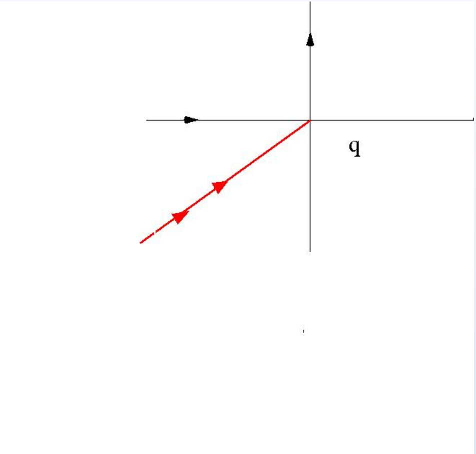

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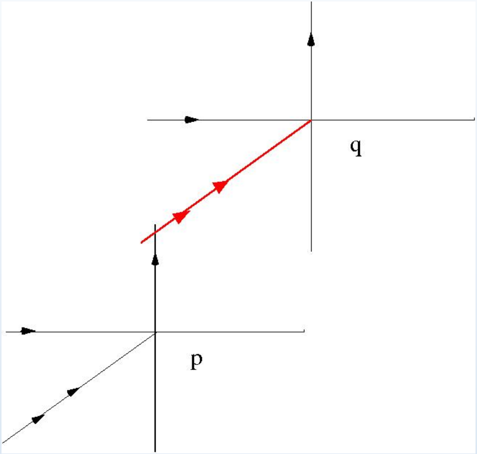

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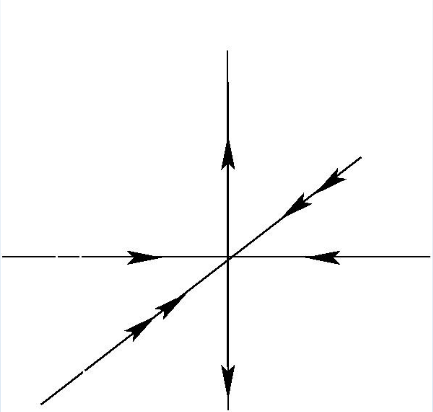

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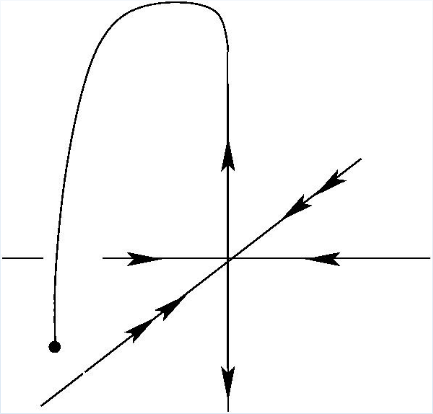

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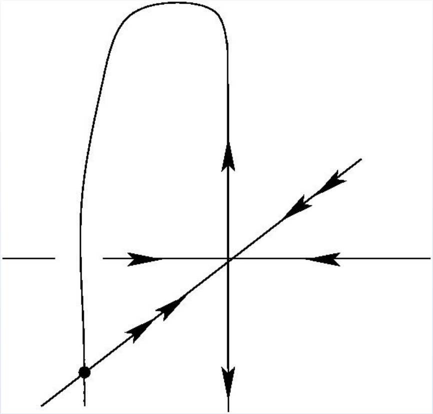

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For any $x \in H_p$ follows that

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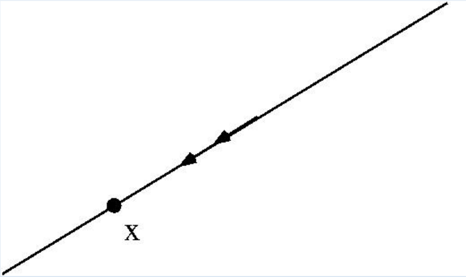

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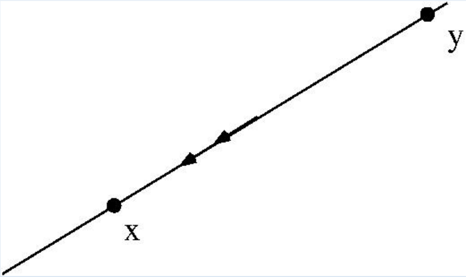

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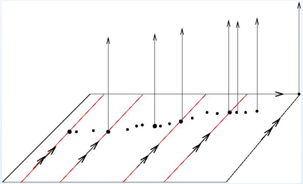

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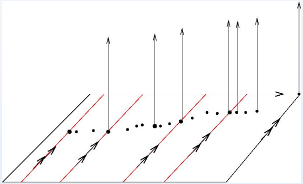

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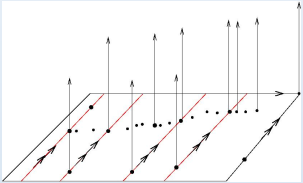

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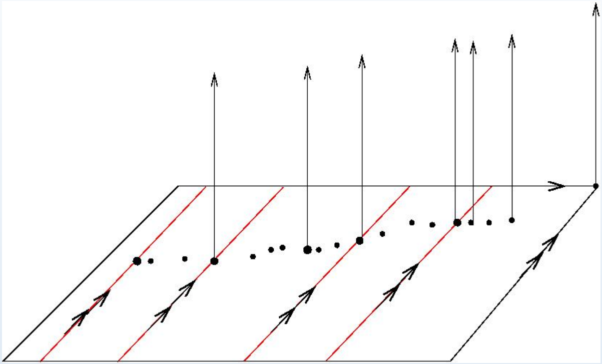

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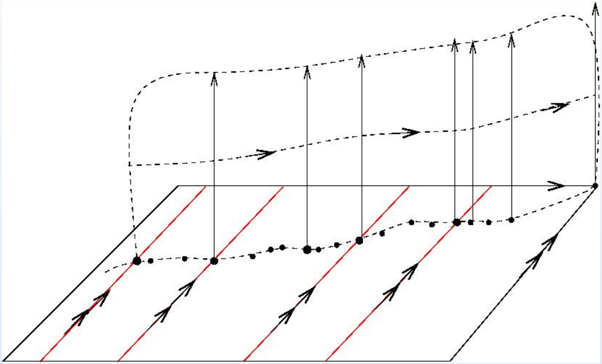

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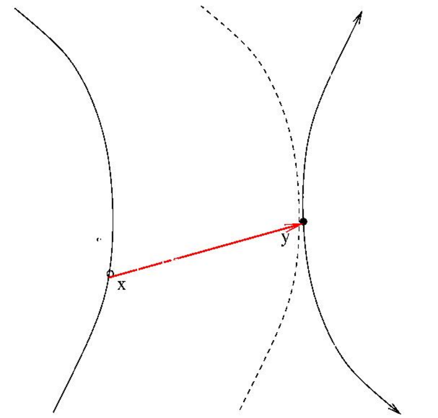

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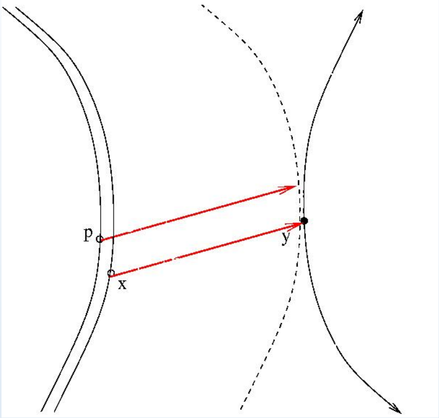

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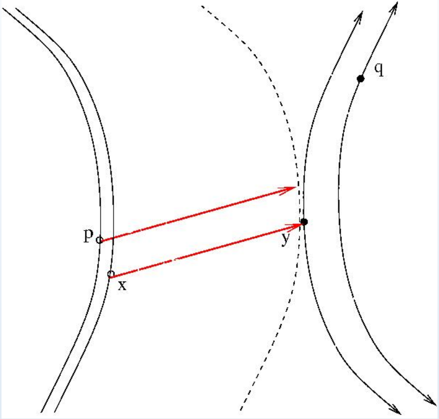

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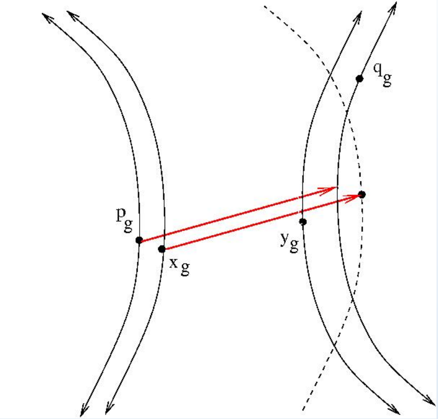

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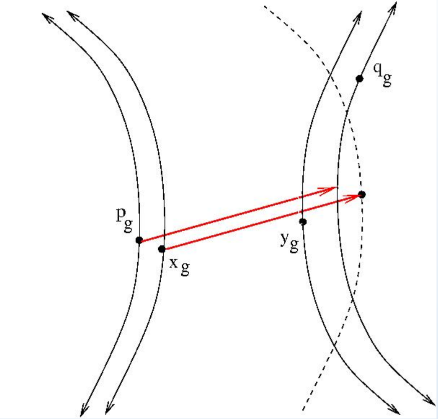

How to do it?

Handling the perturbations.

Moving the unstable manifold of a point respects to the others.

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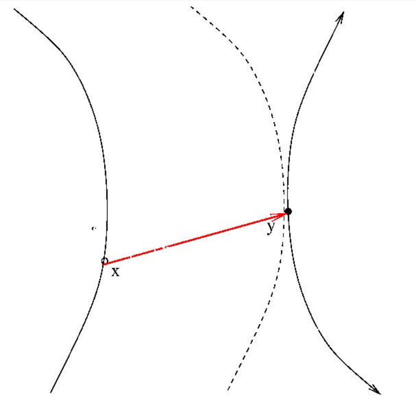

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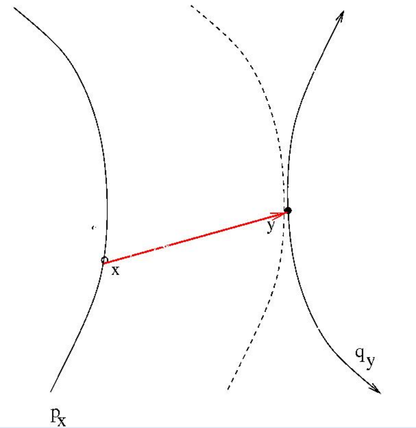

Reasonable if $x \in W^u(p_x), y \in W^u(p_y)$ with p_x, p_y periodics

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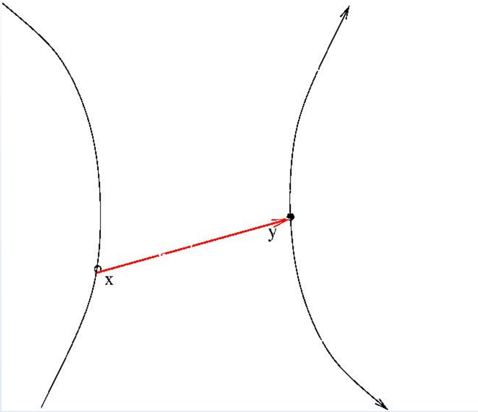

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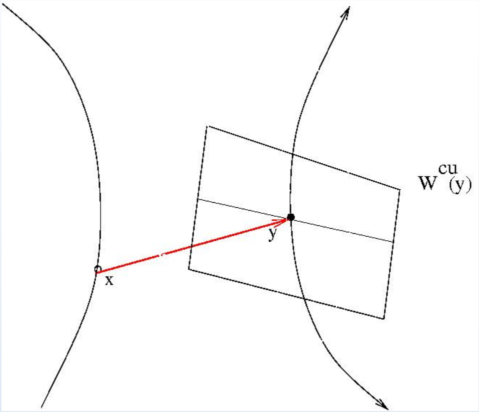

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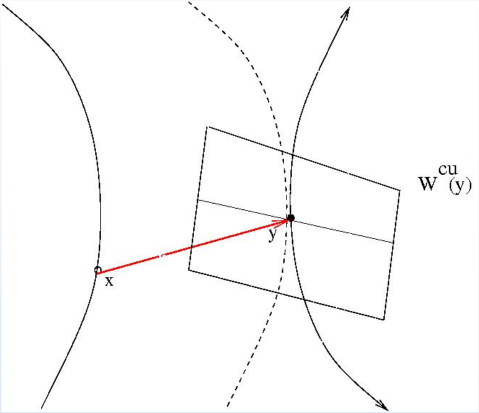

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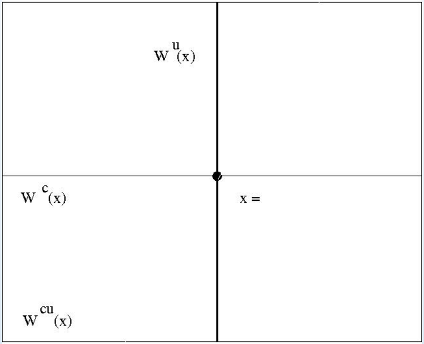

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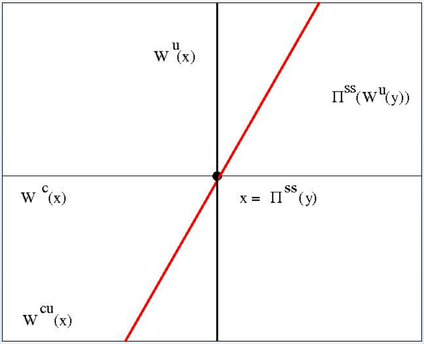

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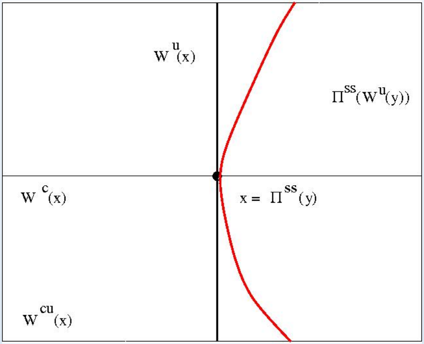

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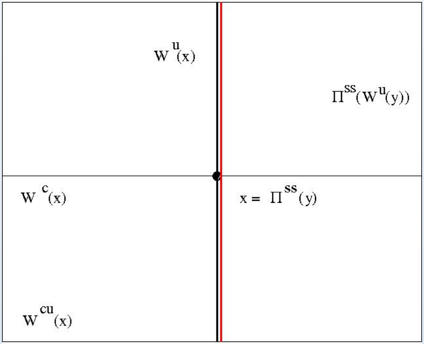

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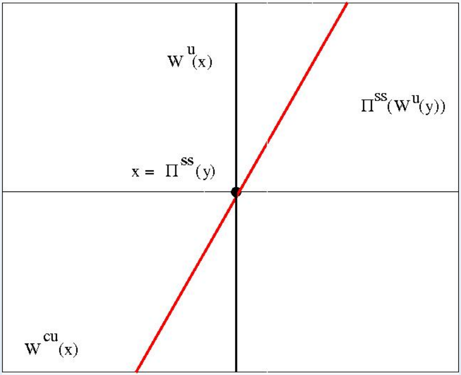

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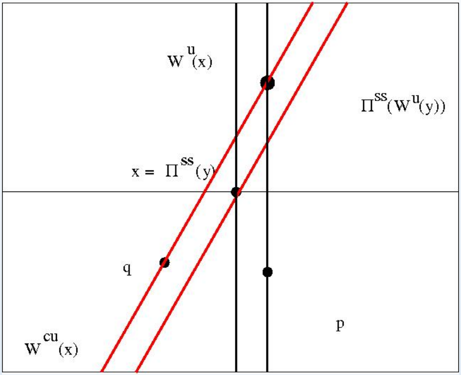

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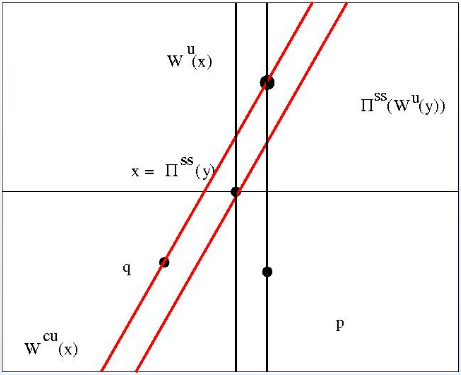

We got the periodic points

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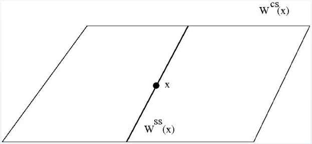

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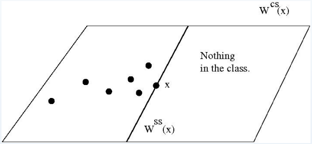

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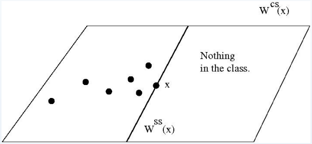

Boundary points belong to unstable manifold of periodic points.

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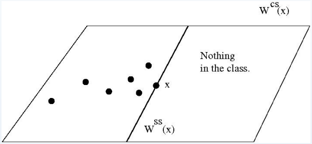

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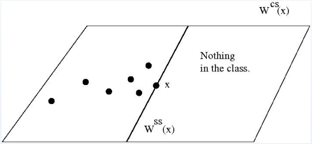

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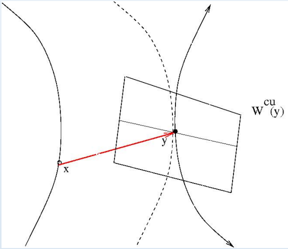

Robust dichotomy: Stable/Unstable.

Robustly, not strong connection and no hyperbolicity:

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Stable case (or joint integrable)

There exists $x, y \in H$ such that $\pi^{ss}(W^u_{\epsilon}(x)) = W^u_{\epsilon}(y)$.

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2. There exists g and $q(g)$ periodic such that

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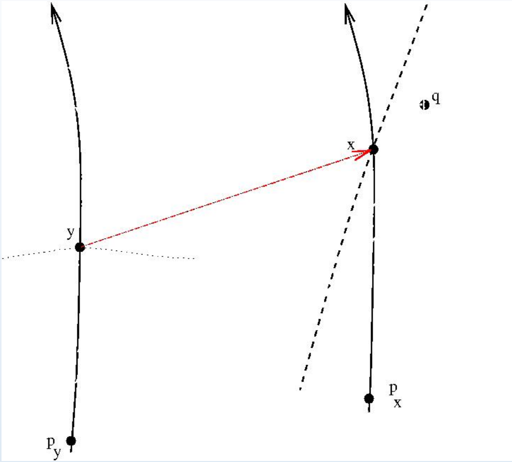

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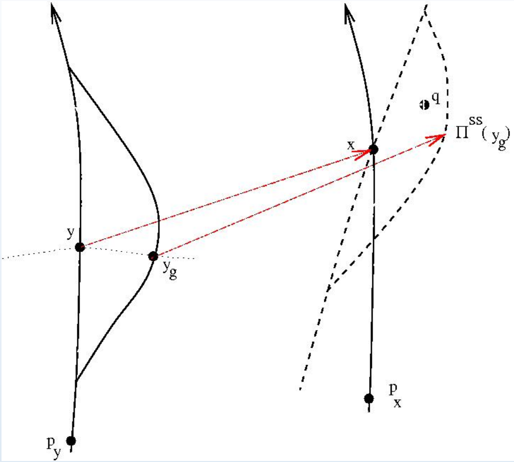

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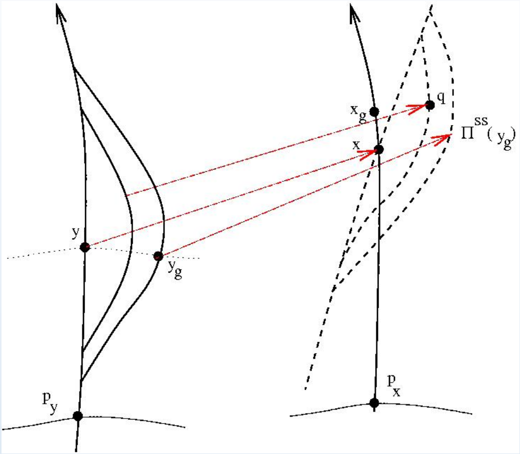

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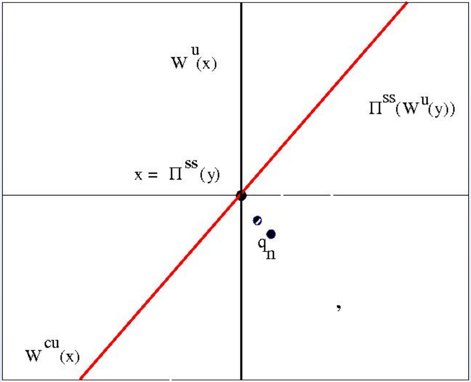

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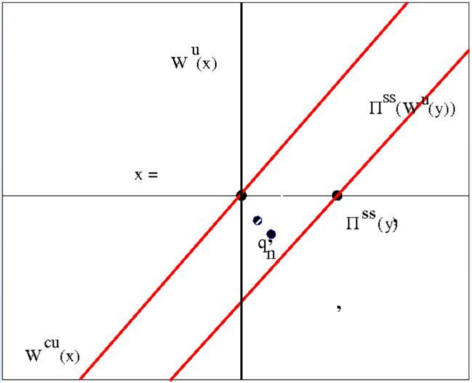

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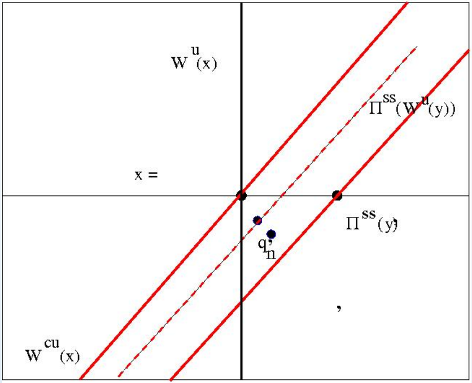

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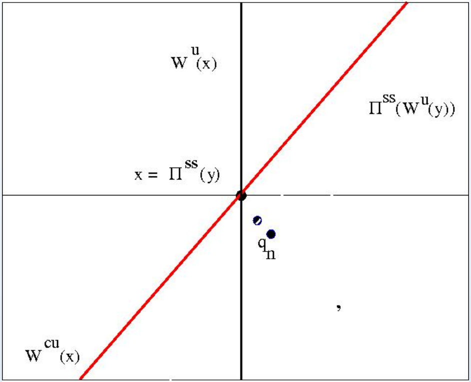

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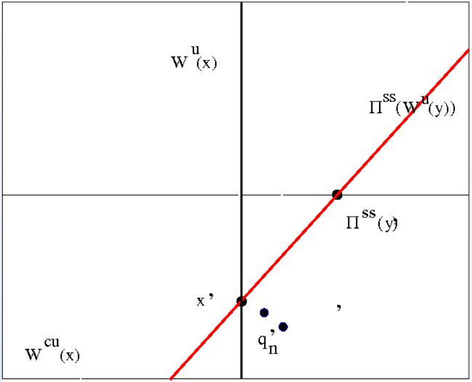

Unstable case. Perturbation.

Recall continuations of points

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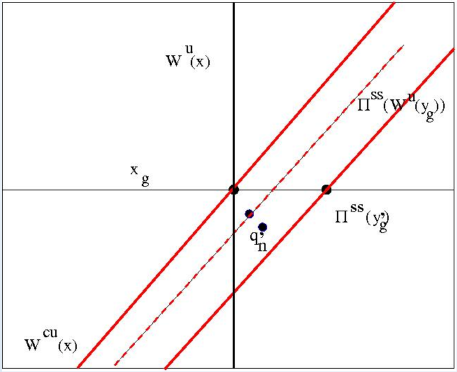

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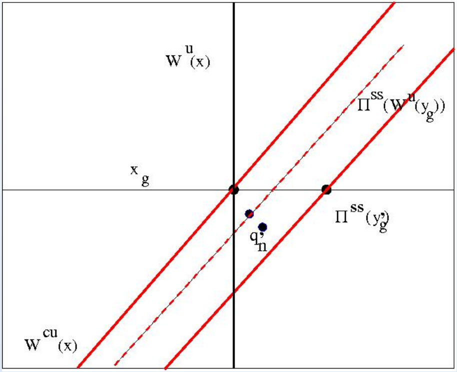

NOT ALWAYS IS POSSIBLE

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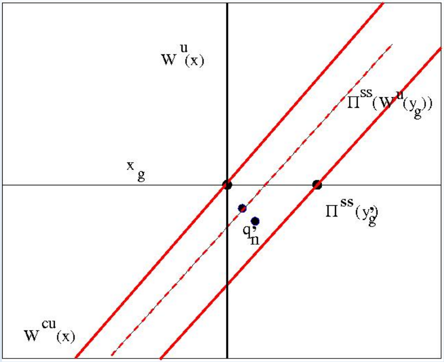

NOT ALWAYS IS POSSIBLE

Long time for the forward iterate of x to visit $B(y)$.

Unstable case. Dichotomy related to return times

$\lambda, \lambda_c, \lambda_u,$

Unstable case. Dichotomy related to return times

$\lambda, \lambda_c, \lambda_u$, N large

Unstable case. Dichotomy related to return times

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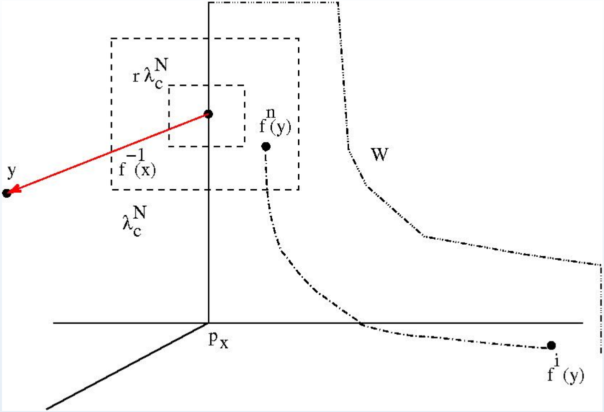
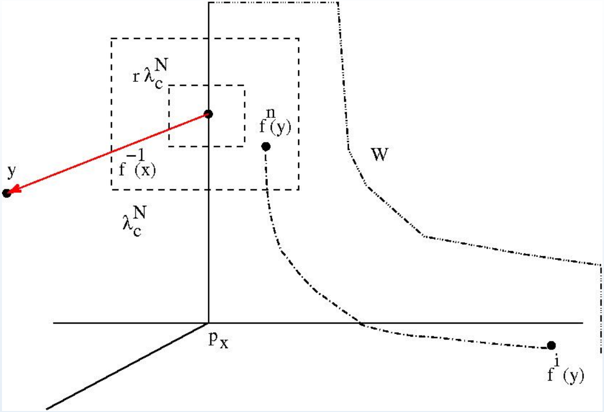
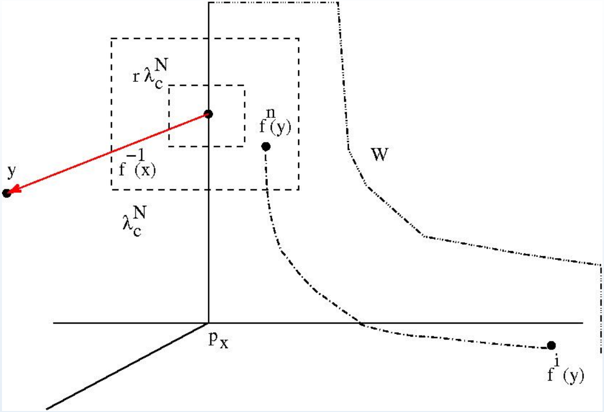


$r\lambda_c^N$
$f^n(y)$
W
y
$f^{-1}(x)$
λ_c^N
p_x
$f^i(y)$



SUBORBIT:$f^i(y)\dots f^n(y) \in W$



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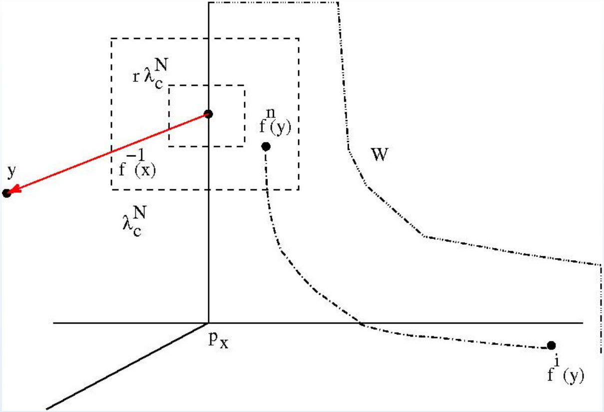
RETURNS HAPPEN ALONG CENTER SUBMANIFOLD



SUBORBIT:$f^i(y) \dots f^n(y) \in W$

RETURNS HAPPEN ALONG CENTER SUBMANIFOLD

$$dist(f^{n-i}(y), W^u(p_x)) \approx \lambda_c^{n-i}$$

Dichotomy about returns

Long time before return

Either $\forall\ r > 0,\ \exists\ N$ large such that

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Relative bounded return

There exists r_0 small, for all large N

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For all large N

$$n < K_0 N$$

$\forall N \quad n < K_0 N$

Get g C^1-close and $q(g) \in H$ periodic such that

$$W^{ss}_{\epsilon}(q(g)) \cap W^u(p_y) \neq \emptyset.$$

$\forall N \quad n < K_0 N$. Deep Return.

Relative bounded return implies Deep return

There is $a > b > 0$, for N large, follows that

1. $n(N) > N$,
2. $f^n(y)$ close to x,
3. $N > a\,n$
4. any suborbit in in W has lenght smaller than $b\,n$.

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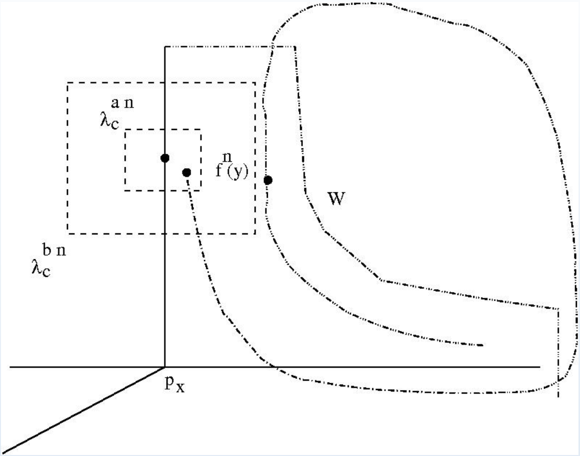

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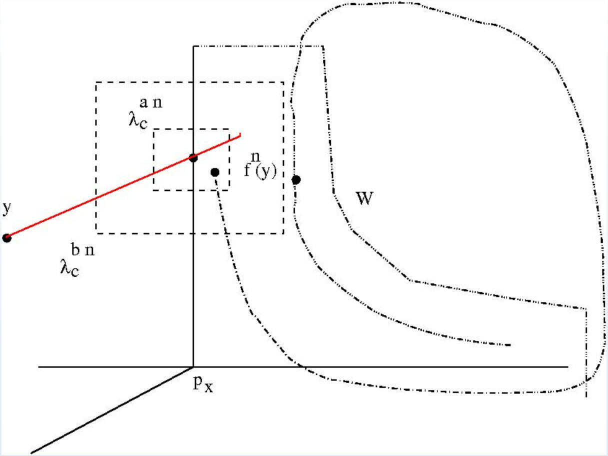

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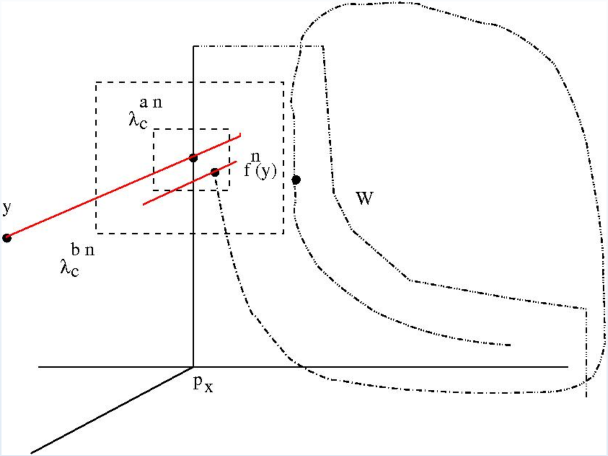

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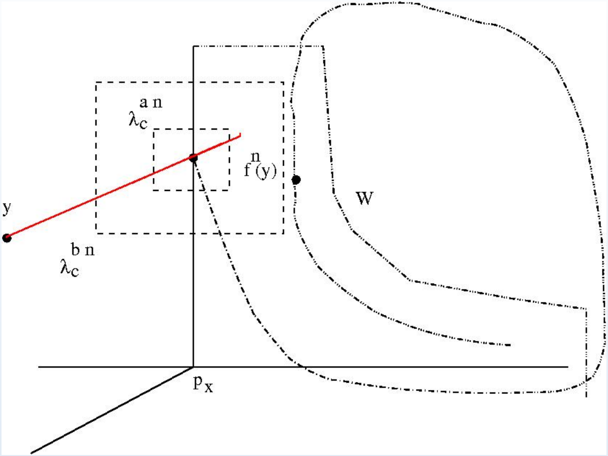

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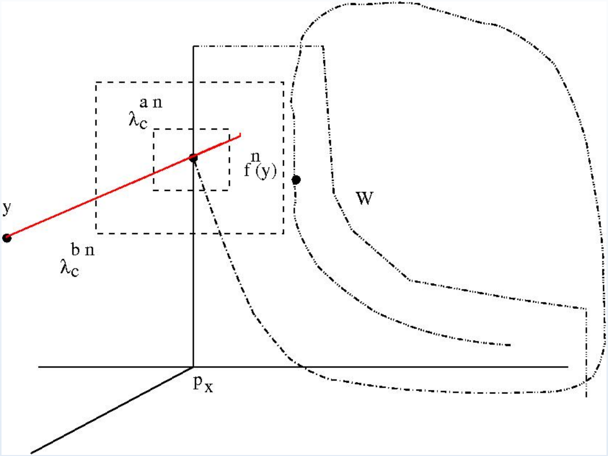

$$\frac{\lambda_c^{an}}{\lambda_c^{bn}} = \lambda_c^{(a-b)n} << 1.$$

$\forall N \quad n = (y, N, r) < K_0 N$. Deep Return.

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There exists $b < \hat{a} < a$ such that

1. $f^n([x, y])$ contained in a neighborhood of size $\lambda_c^{a\,n}$ of x;
2. $dist(f^j([x, y]), x) > \lambda_c^{\hat{a}\,n}$ for any $0 < j < n$.

$$[x, y] = W_r^{ss}(x)$$

$$r = \min\{s : y \in W_s^{ss}(x)\}.$$

$\forall N \quad n < K_0 N$. Deep Return.

There exists g $\epsilon - C^1$-close to f ($\epsilon \approx \lambda_c^{(a-b)n}$):

1. $g^n([x,y]) \subset [x,y]$,
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It is created a periodic point q, such that

$$W^{ss}_\epsilon(q) \cap W^u(p_y) \neq \emptyset.$$

$$q \in H$$

Lemma about Deep Return.

$$\forall N \quad n = (y, N, r) < K_0 N.$$

There is $a > b > 0$, for N large, follows that

1. $n(N) > N$,

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$$m_i < n_i < m_{i+1} < n_{i+1}$$

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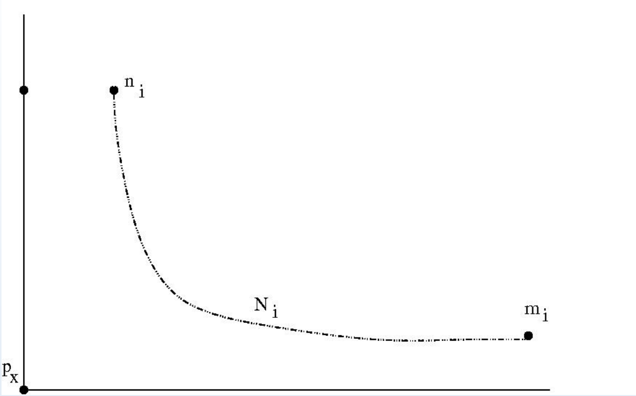
3. if there is a sub-orbit $f^{k_1}(x) \dots f^{k_2}(x)$ with $n_i < k_i < k_2 < m_{i+1}$ then $k_1 - k_2 < N_i$.

$$n_i = n(N_i).$$

Lemma about Deep Return.



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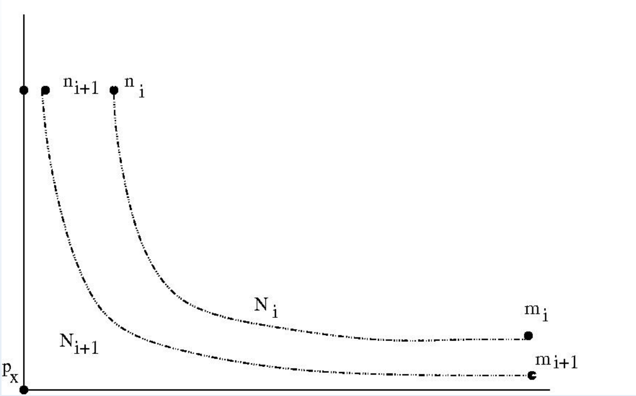

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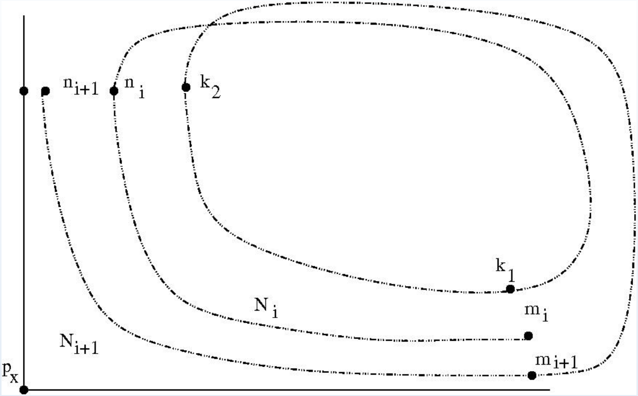

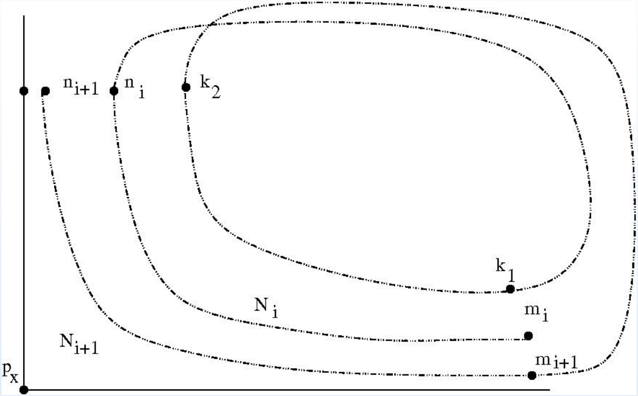

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Lemma about Deep Return ($n < K_0 N$).

$\frac{N_i}{n_i} > \frac{1}{K_0}$. Recall that $n_i = n(N_i)$

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$\frac{N_j}{n_j} < (1+\epsilon)R$ for any j large than j_0

Since $n_j \leq n_i - N_i$ then

$$N_j \leq R(1+\epsilon)n_j \leq R(1+\epsilon)(n_i - N_i) \leq (1+\epsilon)R[1-(1-\epsilon)R]n_i$$

for j larger than j_0

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Since $n_j \leq n_i - N_i$ then

$$N_j \leq R(1 + \epsilon)n_j \leq R(1 + \epsilon)(n_i - N_i) \leq (1 + \epsilon)R[1 - (1 - \epsilon)R]n_i$$

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$b = (1 + \epsilon)[1 - (1 - \epsilon)R] < (1 - \epsilon) = a$ for ϵ small

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$b = (1+\epsilon)[1-(1-\epsilon)R] < (1-\epsilon) = a$ for ϵ small

taking $a = (1-\epsilon)R$ and $b = (1+\epsilon)R[1-(1-\epsilon)R]$ the result holds.

$\exists N \;\; n = (y, N, r) > K_0 N.$

There exists g C^1-close to f such that

$$x_g \notin W^{ss}(y_g).$$

Unstable case. Perturbation.

Goal: $x_g \neq \Pi_g^{ss}(y_g)$

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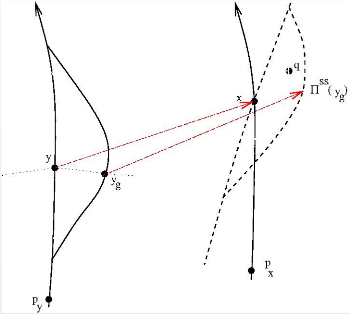

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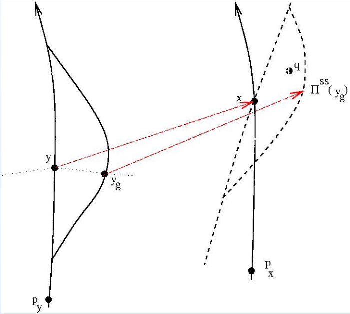

Perturbation: Move x

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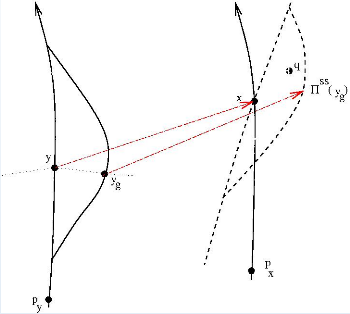

Perturbation: Move x $f^{-1}(x) \to g \circ f^{-1}(x))$

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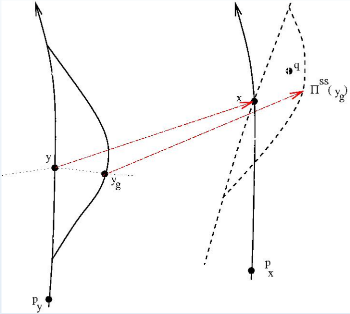

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$$dist(\Pi_g(x_g), y), \quad dist(x_g, x) <<< dist(y, g \circ f^{-1}(x))$$

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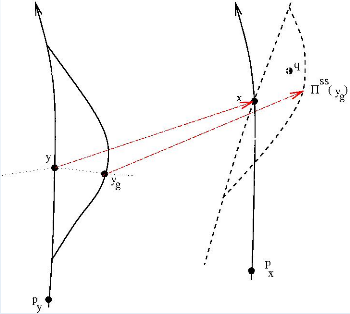

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$dist(\Pi_g(x_g), y), \quad dist(x_g, x) <<< dist(y, g \circ f^{-1}(x))$

Perturbation is concentrated on $B(f^{-1}(x))$

$n > K_0[N + \log(\frac{1}{r})]$

$$x_g \neq \Pi_g^{ss}(y_g)$$

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Estimating: $dist(x_g, g(f^{-1}(x)))$

Estimating: $dist(y_g, y)$

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n, return time to V.

$n > K_0[N + \log(\frac{1}{r})]$

Estimating: $dist(\Pi_f^{ss}(y_g), \Pi_f^{ss}(y))$

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Estimating: $dist(\Pi^{ss}_f(y_g), \Pi^{ss}_f(y))$ $\quad dist(\Pi^{ss}_f(y_g), \Pi^{ss}_g(y_g))$

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Estimating: $dist(\Pi^{ss}_f(y_g), \Pi^{ss}_f(y))$ $dist(\Pi^{ss}_f(y_g), \Pi^{ss}_g(y_g))$

Π^{ss}_f, Π^{ss}_g are Holder; estimate $dist(\Pi^{ss}_g, \Pi^{ss}_f)$

$n > K_0[N + \log(\frac{1}{r})]$

Estimating: $dist(\Pi_f^{ss}(y_g), \Pi_f^{ss}(y))$ $dist(\Pi_f^{ss}(y_g), \Pi_g^{ss}(y_g))$

Π_f^{ss}, Π_g^{ss} are Holder; estimate $dist(\Pi_g^{ss}, \Pi_f^{ss})$

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$$dist(\Pi_f^{ss}(y_g), \Pi_f^{ss}(y)) < dist(y_g, y)^{\alpha_s} < [r\lambda_c^N \lambda_u^{-n}]^{\alpha_s}$$

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$$dist(\Pi_g^{ss}(z), \Pi_f^{ss}(z)) < \lambda^{n_z}$$

$n > K_0[N + \log(\frac{1}{r})]$

Estimating: $dist(\Pi_f^{ss}(y_g), \Pi_f^{ss}(y))$ $\quad dist(\Pi_f^{ss}(y_g), \Pi_g^{ss}(y_g))$

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λ, constant of domination, n_z return time to V

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Π_f^{ss}, Π_g^{ss} are Holder; estimate $dist(\Pi_g^{ss}, \Pi_f^{ss})$

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λ, constant of domination, n_z return time to V

By the election of K_0 and $n > K_0[N + \log(\frac{1}{r})]$

Everything works

Joint integrable case/Stable case.

$$\Pi^{ss}(W^u_{loc}(y)) = W^u_{loc}(x)$$

Joint integrable case/Stable case.

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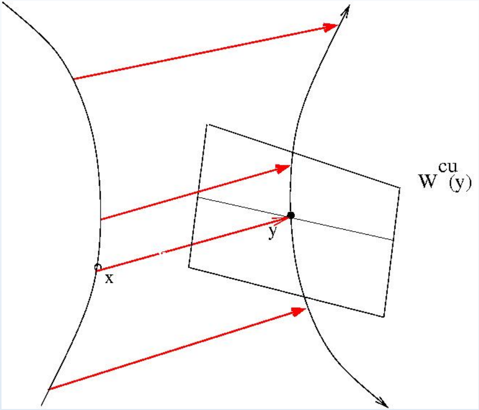

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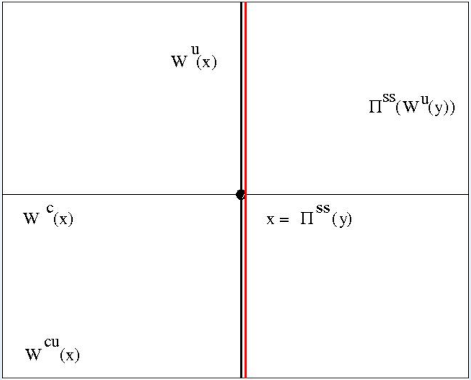

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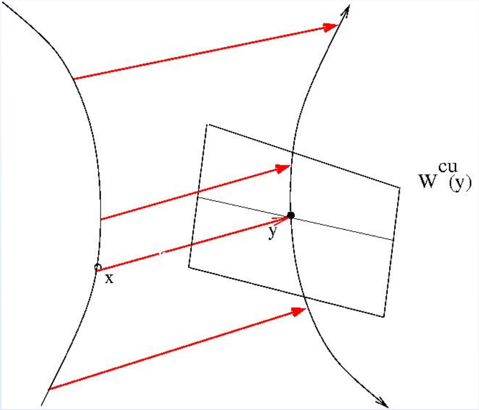

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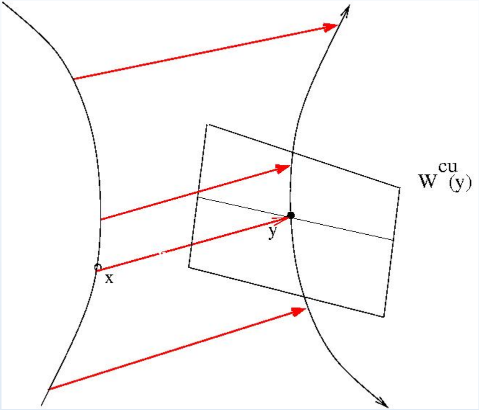

We can not assume that x or y belongs to unstable of periodic pts.

Joint integrable case/Stable case.

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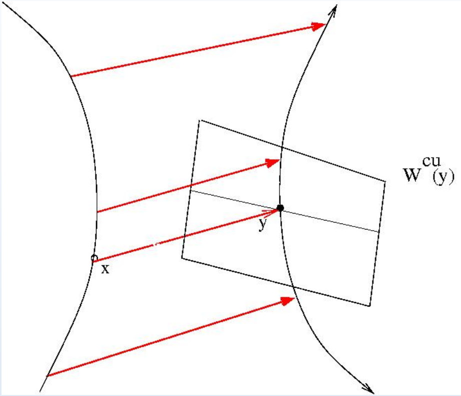
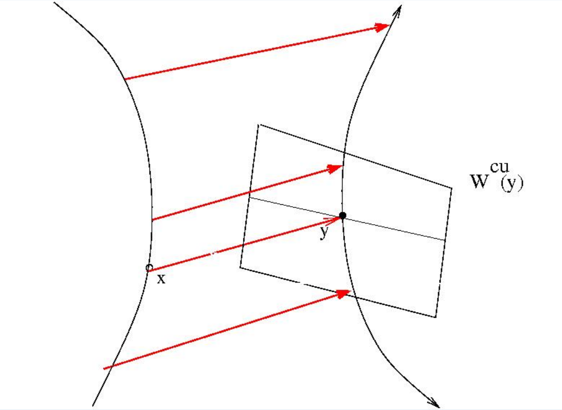


We can not assume that x or y belongs to unstable of periodic pts.

If they are, we argue as before

Joint integrable case/Stable case.



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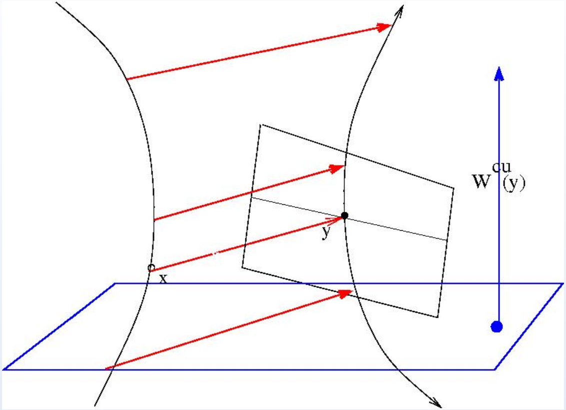

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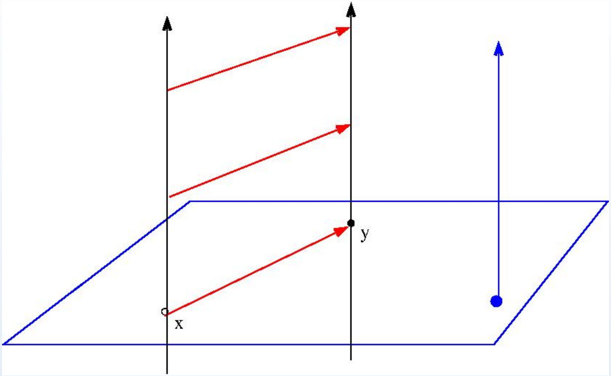

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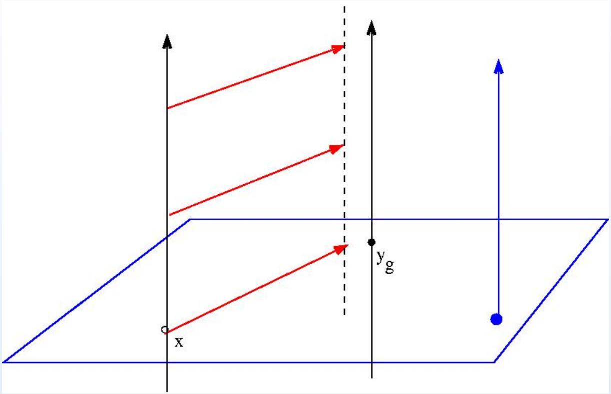

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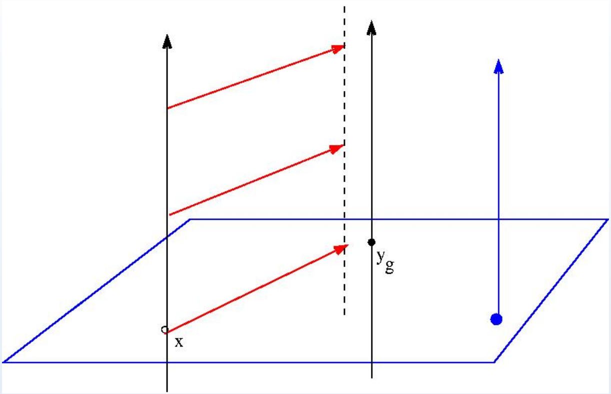
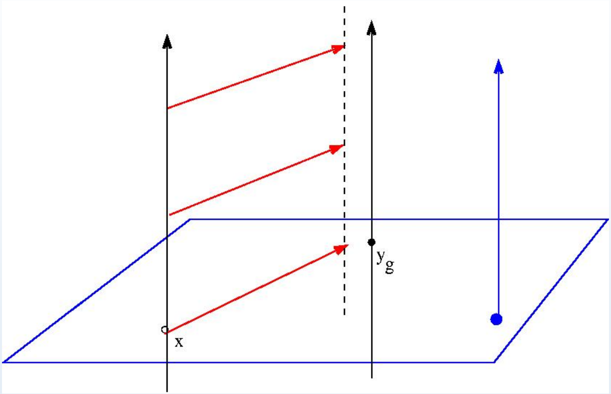


If $\Pi^{ss}(x_g) \neq y_g$ then is done!

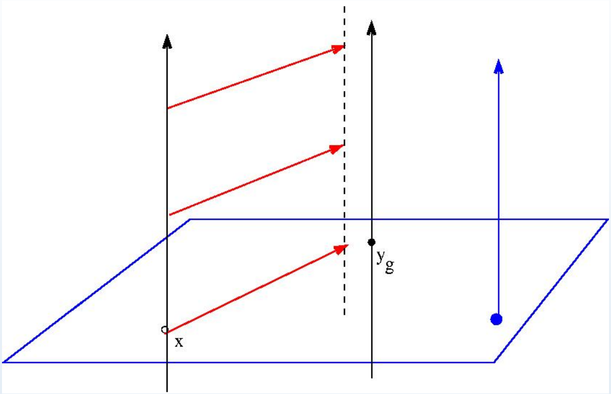
Joint integrable case/Stable case.



If $\Pi^{ss}(x_g) \neq y_g$ then is done!

If x and y are not in the unstable of periodic points,

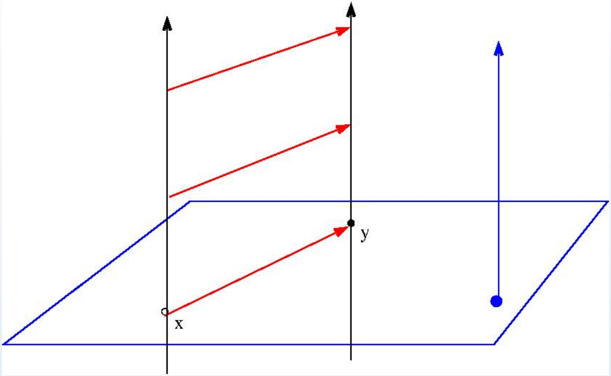
Joint integrable case/Stable case.



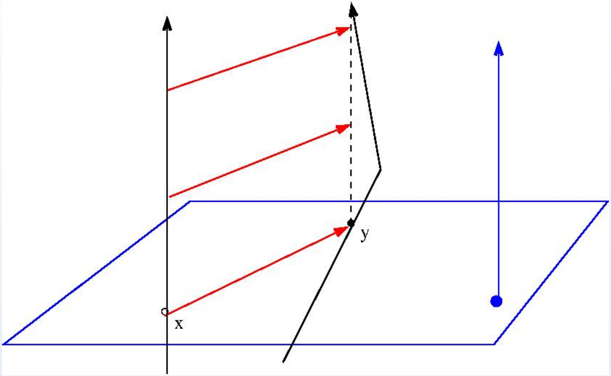
If $\Pi^{ss}(x_g) \neq y_g$ then is done!

If x and y are not in the unstable of periodic points, accumulated by both sides by periodic points.

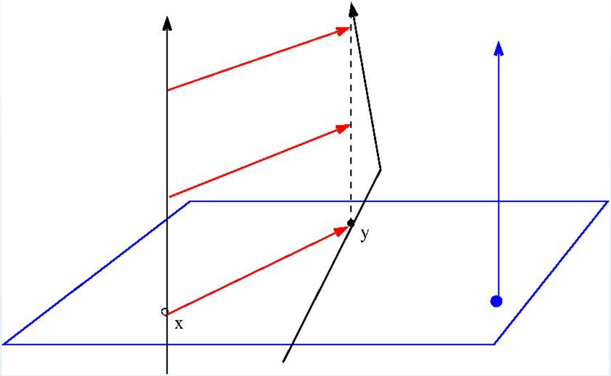
Joint integrable case/Stable case. Assuming $\Pi^{ss}(x_g) \neq y$



Joint integrable case/Stable case. Assuming $\Pi^{ss}(x_g) \neq y$

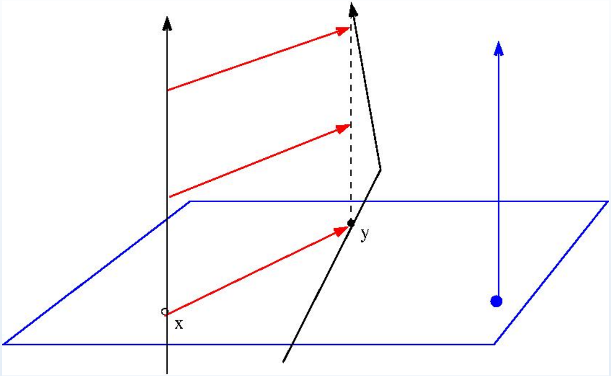


Joint integrable case/Stable case. Assuming $\Pi^{ss}(x_g) \neq y$



Break joint integrability.

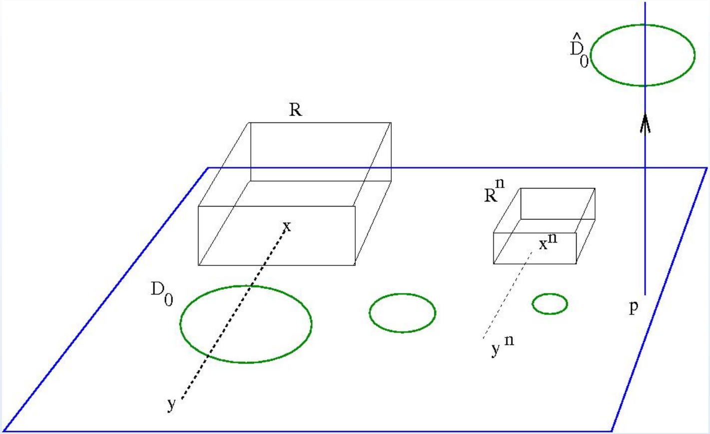
Joint integrable case/Stable case. Assuming $\Pi^{ss}(x_g) \neq y$



Break joint integrability.

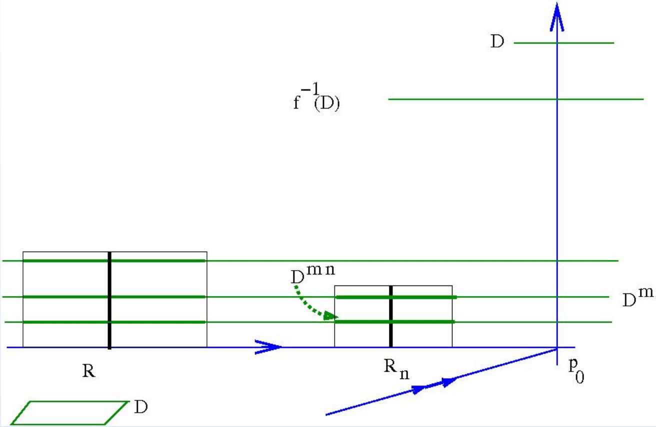
Enough to get a connection.

How to break joint integrability. Type of perturbations.



C^1 –Linearizing coordinates $W^s_\epsilon(p), W^{ss}_\epsilon \subset W^s_\epsilon(p), D, W^u_\epsilon(x)$

How to break joint integrability. Type of perturbations.



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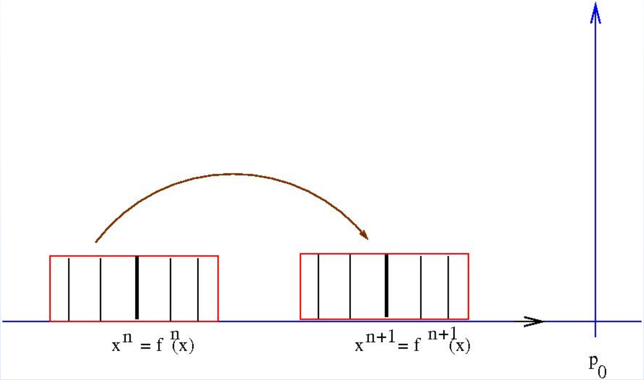

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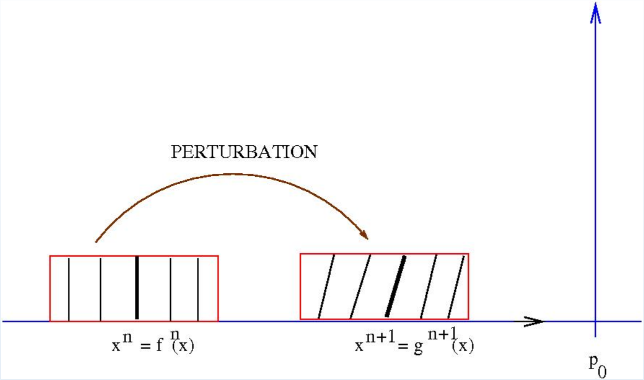

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Support in $R_n(\eta^u) = f^n(R_0) \cap \{|\hat{z}| < \eta^u\}$.

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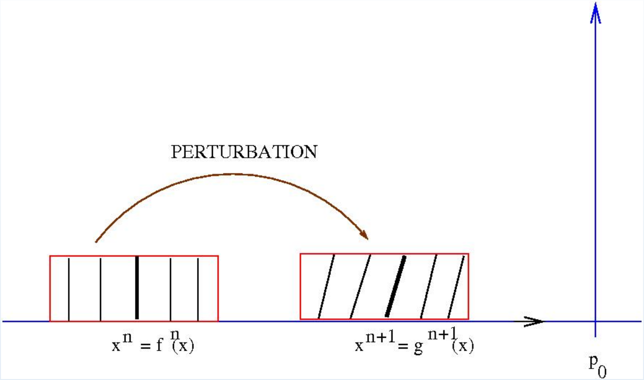
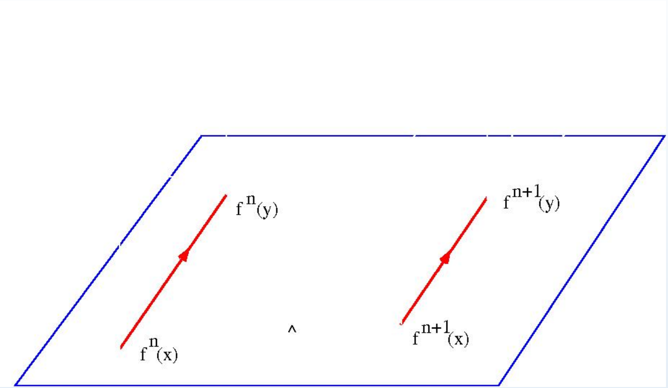


C^1 −Linearizing coordinates $W^s_\epsilon(p), W^{ss}_\epsilon \subset W^s_\epsilon(p), D, W^u_\epsilon(x)$

Support in $R_n(\eta^u) = f^n(R_0) \cap \{|\hat{z}| < \eta^u\}$.

For any $z \in R_n(\eta^u) \cap W^s_\epsilon(p)$ $Dg : (0, 0, v^u) \to (0, \eta_0, v^u)$

How to break joint integrability. Type of perturbations.

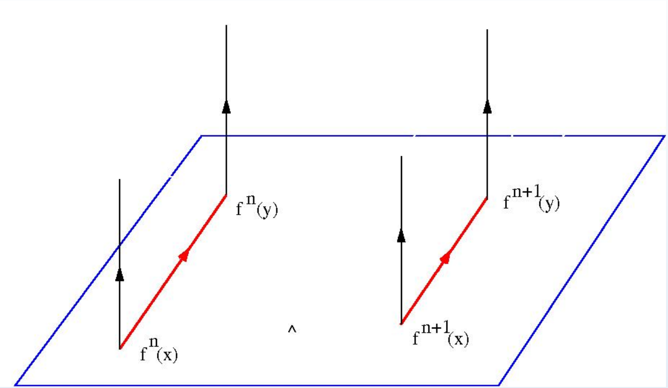


$$f^n(x), f^n(y), R_n(f^n(y), \eta^u),$$

perturbation support in R_n (all vector there is affected), $D_{n,m}$ not affected.

$W^u(g^{n+1}(x))$ is the same, $W^u(g^{n+1}(y))$ is tilted with angle η_0

How to break joint integrability. Type of perturbations.



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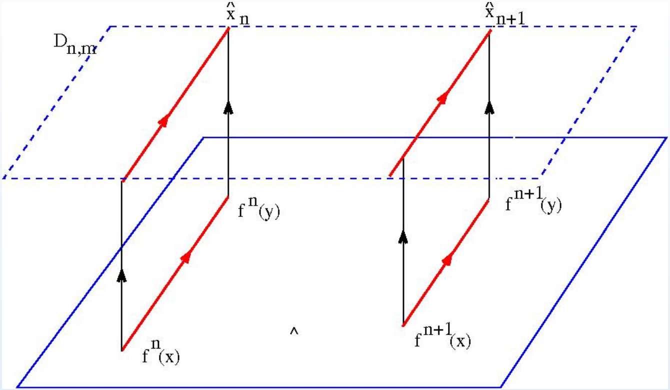

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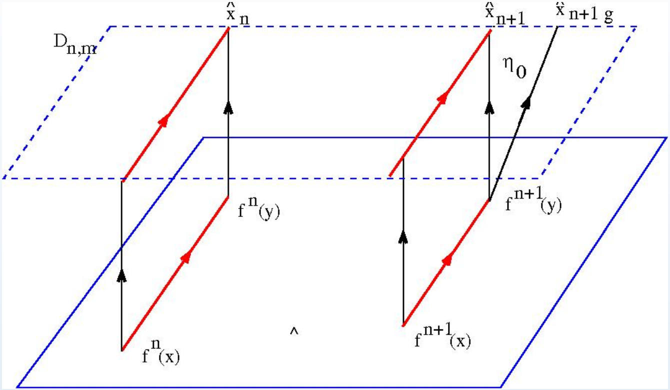

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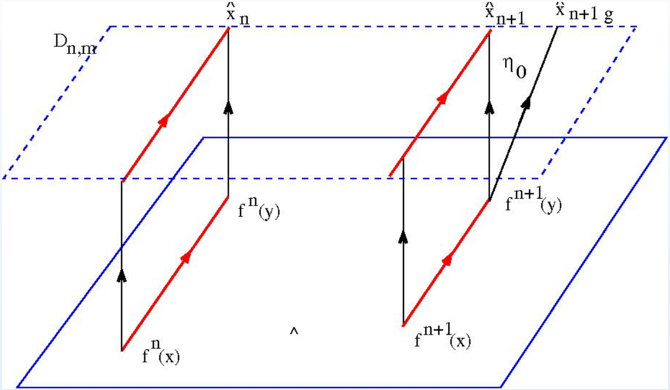

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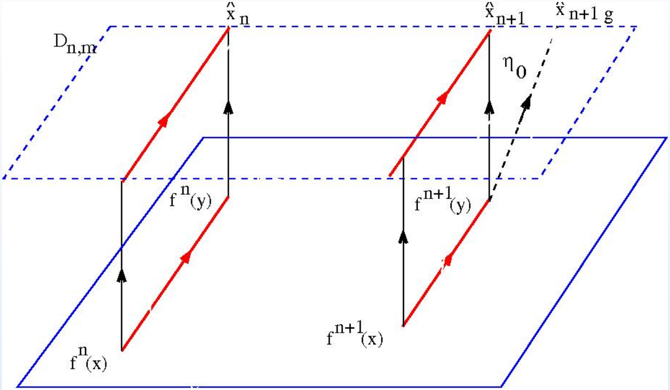
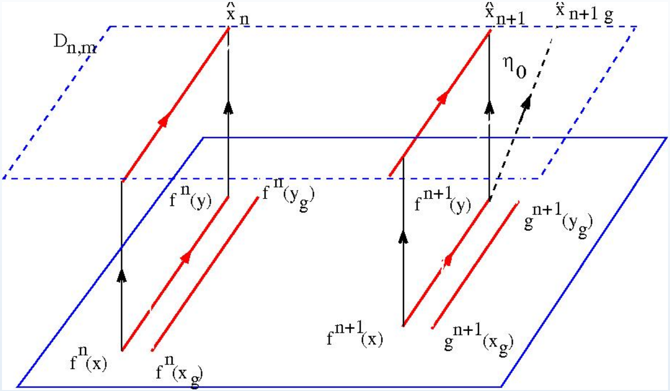
How to break joint integrability. Problems



The continuations x_g, y_g move. So, $g^n(x_g), g^n(y_g)$

How to break joint integrability. Problems



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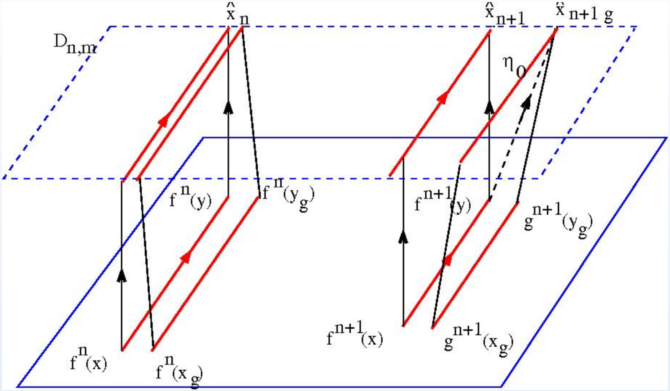
How to break joint integrability. Problems



The continuations x_g, y_g move. So, $g^n(x_g), g^n(y_g)$

The unstable of $g^n(x_g), g^n(y_g)$ are not necessarily vertical.

Changes in $g^n(x_g), g^n(y_g)$ can upset perturbation.

How to break joint integrability. Control

Unstable subbundle is Holder

How to break joint integrability. Control

Unstable subbundle is Holder

$$dist(f^n(x), g^n(x_g)) = dist(f^n(x), f^n(x_g)) < C_0\lambda_s^n$$

How to break joint integrability. Control

Unstable subbundle is Holder

$$dist(f^n(x), g^n(x_g)) = dist(f^n(x), f^n(x_g)) < C_0\lambda_s^n$$

$Slope(E_f^u(f^n(x)), E_g^u(g^n(x_g)))$

How to break joint integrability. Control

Unstable subbundle is Holder

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Unstable subbundle is Holder

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k_x first backward visit to $R_n(\eta^u)$

How to break joint integrability. Control

Unstable subbundle is Holder

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Unstable subbundle is Holder

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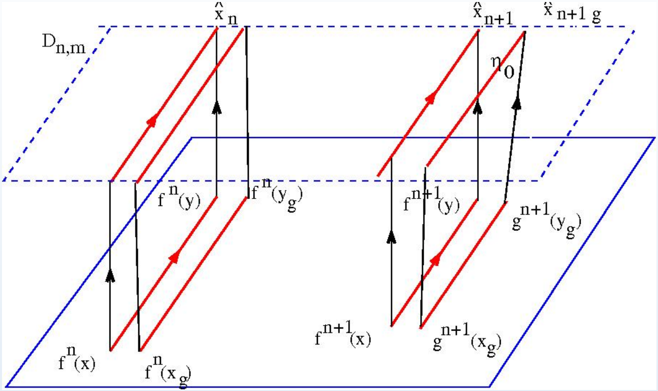
How to break joint integrability. Concluding



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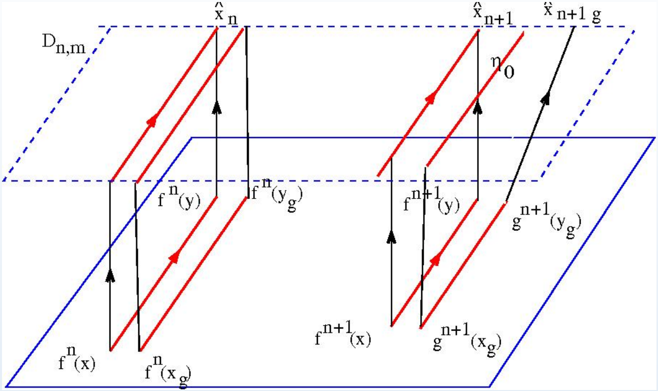

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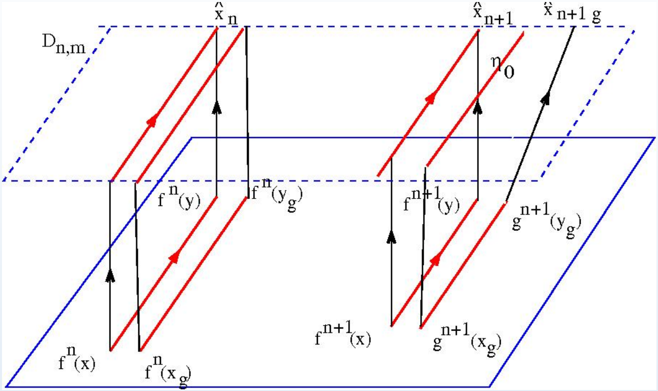

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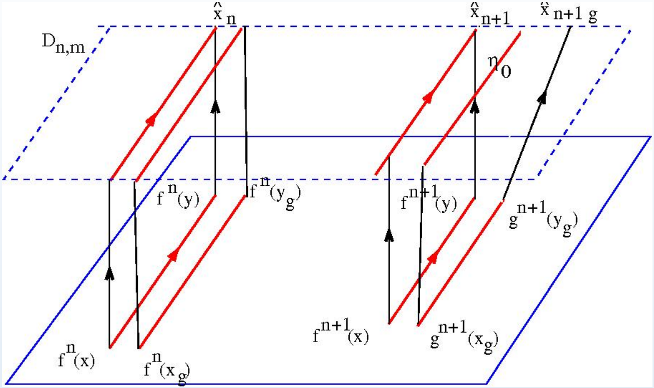

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If W^{cs} is not totally disconnected, then cycle

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It is needed C^2; so, problem. Look for another approach.

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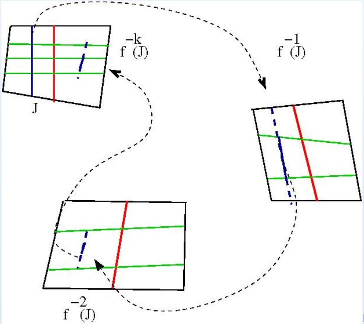

$E^s \oplus E^{cu}$ Using the Markov partition. Summability

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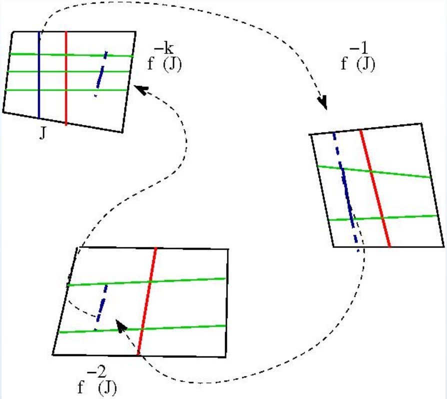

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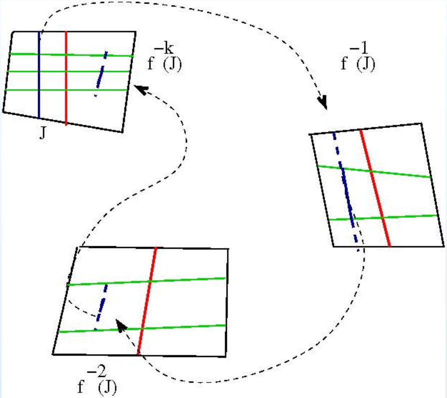

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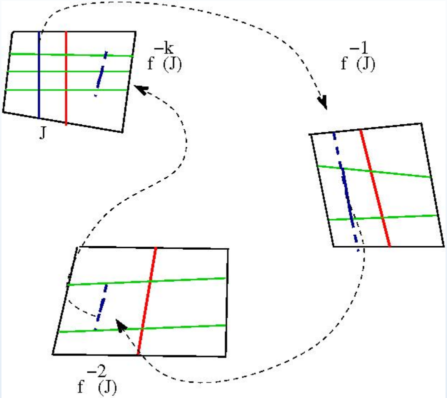

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Using hyperbolic times, summability is recovered

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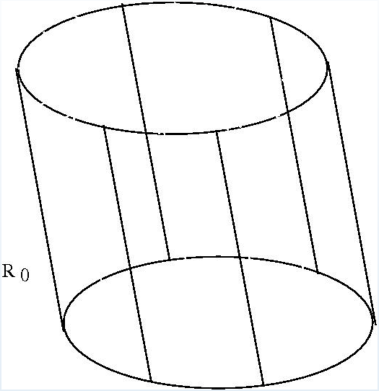

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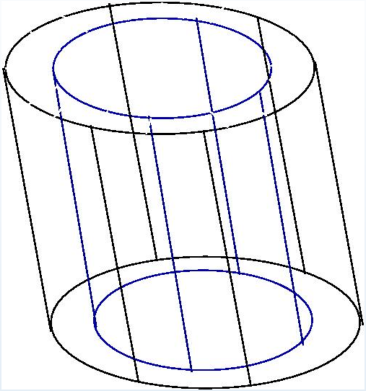

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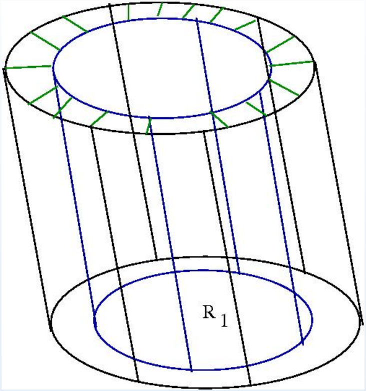

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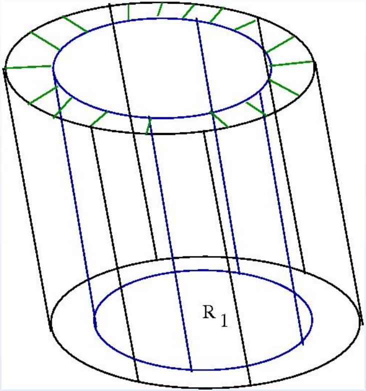

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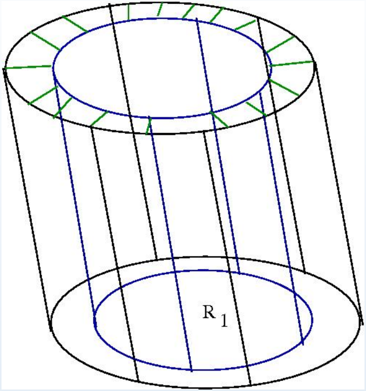

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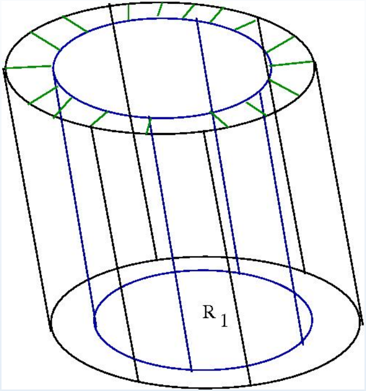
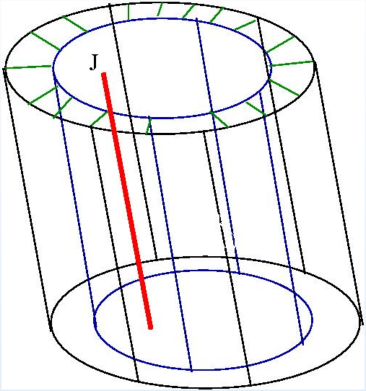


$$[R_0 \setminus R_1] \cap H = \emptyset$$

W^{cs} is thin trapped

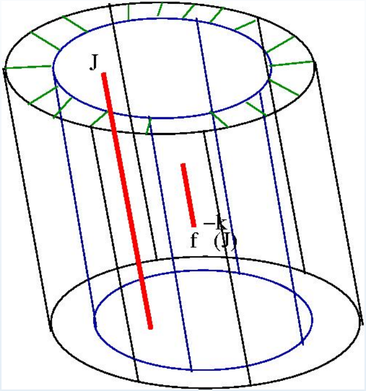
$W^{cs} \cap H$ is totally disconnected

$E^s \oplus E^{cs} \oplus E^{cu}$



$J \subset R_1$

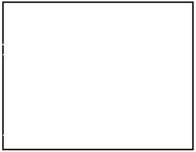
$E^s \oplus E^{cs} \oplus E^{cu}$



$$J \subset R_1$$

either $f^{-k}(J) \cap R_0 = \emptyset$ or $f^{-k}(J) \subset R_1$

$E^s \oplus E^{cs} \oplus E^{cu}$



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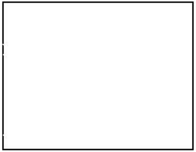

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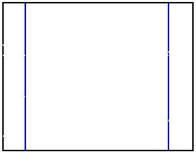

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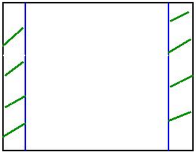

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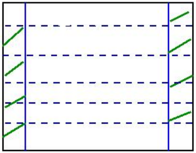

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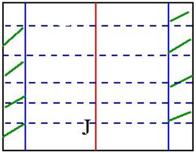
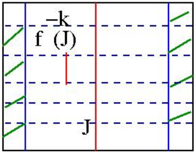


$$J \subset R_0; \quad f^{-k}(J) \subset R; \quad f^{-m}(J) \cap R = \emptyset$$



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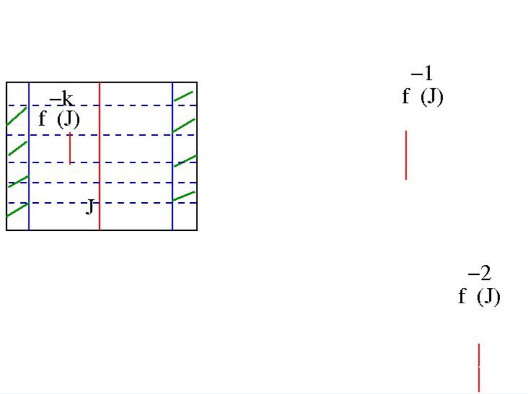
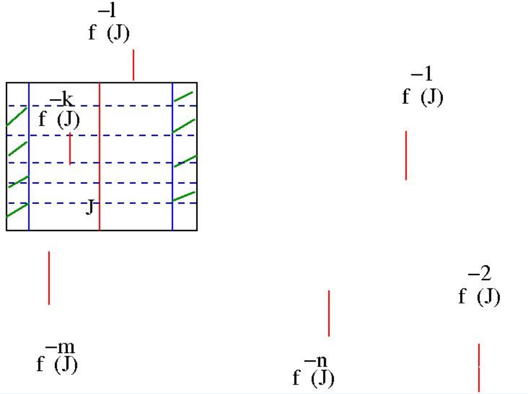


$$J \subset R_0; \quad f^{-k}(J) \subset R; \quad f^{-m}(J) \cap R = \emptyset$$

$$\Sigma_{n=0}^{k} \ell(f^{-n}(J)) < K_0$$



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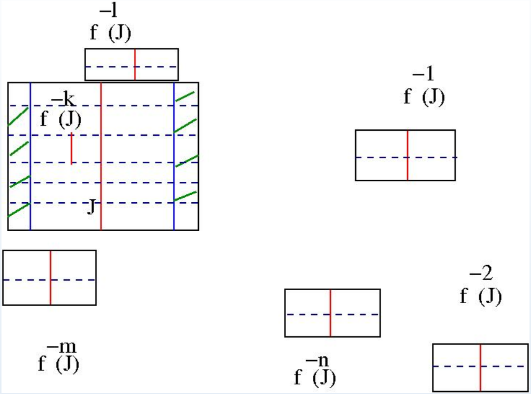

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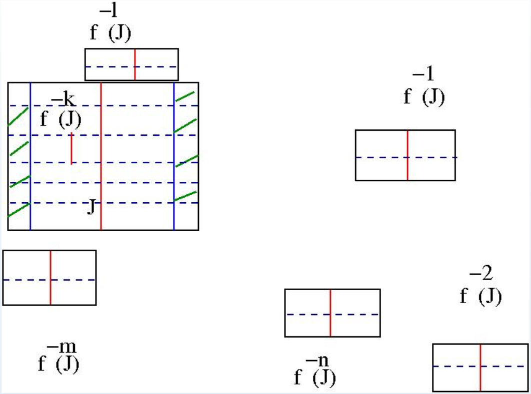

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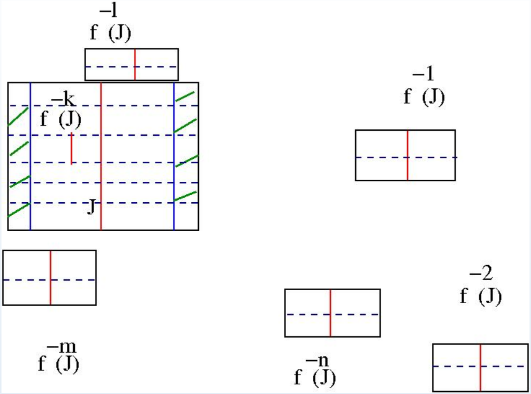

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$\Sigma_{n=0}^{k} vol(B_\gamma(f^{-n}(J))) < K_0$ implies $\Sigma_{n=0}^{k} \ell(f^{-n}(J)) < K_0$

Why $B_\gamma(f^{-n}(J)) \cap B_\gamma(f^{-m}(J)) = \emptyset$?

$E^s \oplus E^{cs} \oplus E^{cu}$

Why $B_\gamma(f^{-n}(J)) \cap B_\gamma(f^{-m}(J)) = \emptyset$?CAREFULL

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- It is only taking the maximal sequences n_i:

$E^s \oplus E^{cs} \oplus E^{cu}$

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$$|Df^j_{E^{cs}(f^{-n_i}(x))}| < \lambda^j, \ j = 1 \dots n_i \quad \lambda < 1.$$

$E^s \oplus E^{cs} \oplus E^{cu}$

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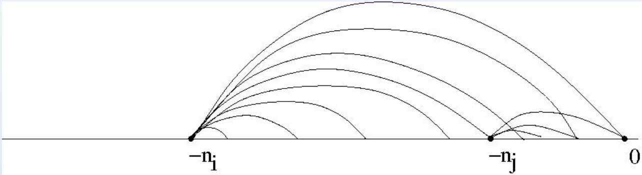

$E^s \oplus E^{cs} \oplus E^{cu}$

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Properties:

$E^s \oplus E^{cs} \oplus E^{cu}$

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Properties:

- For any $y \in f^{-n_i}(J)$, $f^j(W^{cs}_\gamma(y)) \subset W^{cs}_{\lambda^j \gamma}(f^j(y))$

$E^s \oplus E^{cs} \oplus E^{cu}$

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$E^s \oplus E^{cs} \oplus E^{cu}$

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$$\ell(I^{cu}_{B_{n_i}}) \approx \ell(f^{-n_i}(J)).$$

$E^s \oplus E^{cs} \oplus E^{cu}$

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- for any n_j follows that

$E^s \oplus E^{cs} \oplus E^{cu}$

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- For any $y \in f^{-n_i}(J)$, $f^j(W^{cs}_\gamma(y)) \subset W^{cs}_{\lambda^j\gamma}(f^j(y))$
- For any $B_{n_i} = B_\gamma(f^{-n_i}(J)$ and $I^{cu}_{B_{n_i}}$ follows that

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- for any n_j follows that

$$|Df^{-i}_{E^{cu}(f^{-n_j}(x))}| < \lambda^i, \quad 0 \leq i \leq n_{j+1} - n_j$$

$E^s \oplus E^{cs} \oplus E^{cu}$

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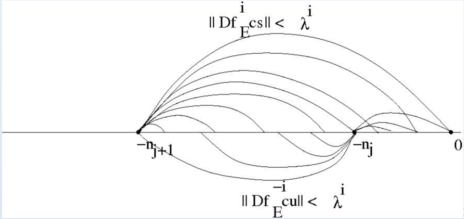

$B_{n_i} \cap B_{n_j} = \emptyset$ at Pliss's iterates

$E^s \oplus E^{cs} \oplus E^{cu}$

$B_{n_i} \cap B_{n_j} = \emptyset$ at Pliss's iterates

$$f^{n_i}(B_{n_i}) \subset R_0, \;\; diameter(f^{n_i}(B_{n_i})) < \frac{diameter(R_0 \backslash R_1)}{2}$$

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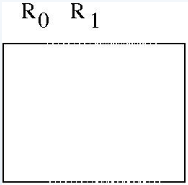

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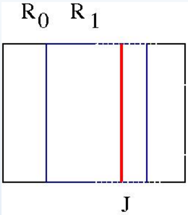

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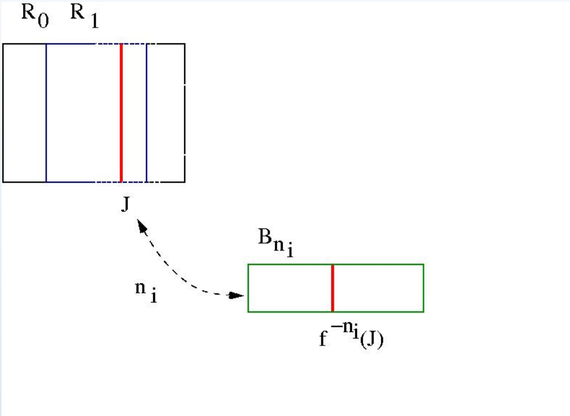

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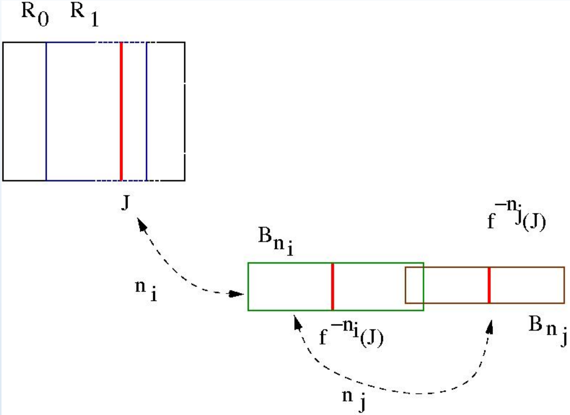

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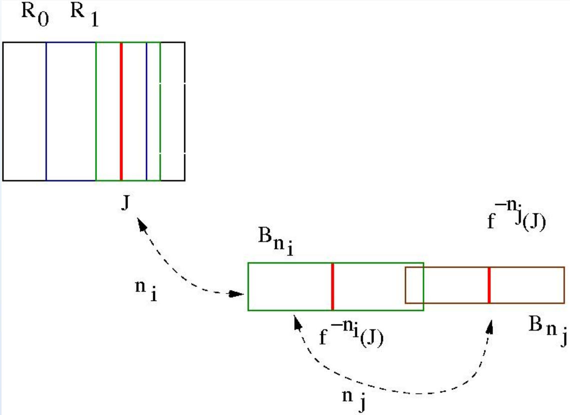

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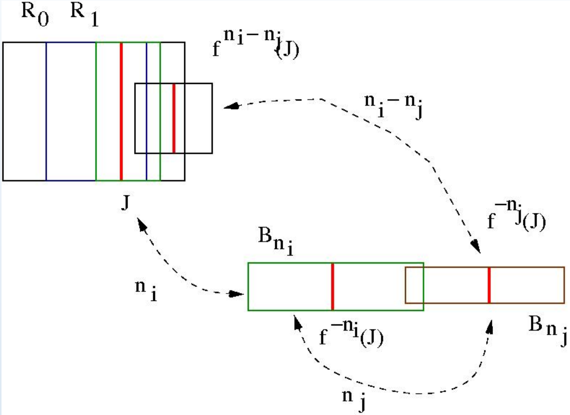

$E^s \oplus E^{cs} \oplus E^{cu}$

Putting together

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$E^s \oplus E^{cs} \oplus E^{cu}$

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Good distortion at Pliss's iterates

$E^s \oplus E^{cs} \oplus E^{cu}$

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Good distortion at Pliss's iterates

Summability of boxes at Pliss's iterates

$E^s \oplus E^{cs} \oplus E^{cu}$

Putting together

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Summability of boxes at Pliss's iterates

$$\Sigma_{n_j} \ell(f^{-n_j}(J)) < K_0$$

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Good distortion at Pliss's iterates

Summability of boxes at Pliss's iterates

$\Sigma_{n_j} \ell(f^{-n_j}(J)) < K_0$

Contraction along centerunstable in between Pliss's iterates.

Putting together

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Whole summability $\Sigma_n \ell(f^{-n}(J)) < K_0$

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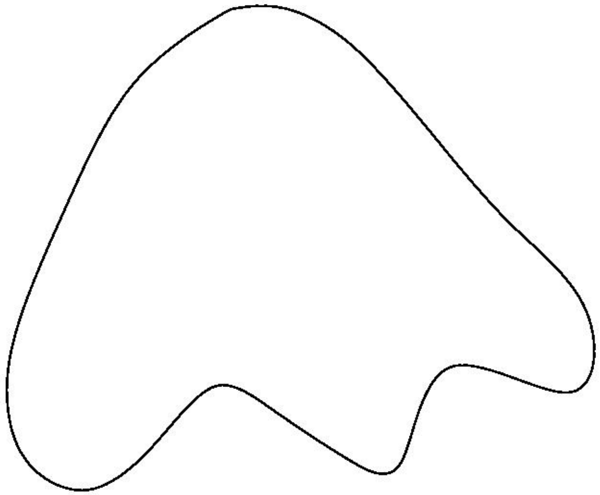
Whole summability $\quad \Sigma_n \ell(f^{-n}(J)) < K_0$

This should be enough as idea of the proof

Recalling: Far from bifurcations, NICE PICTURE



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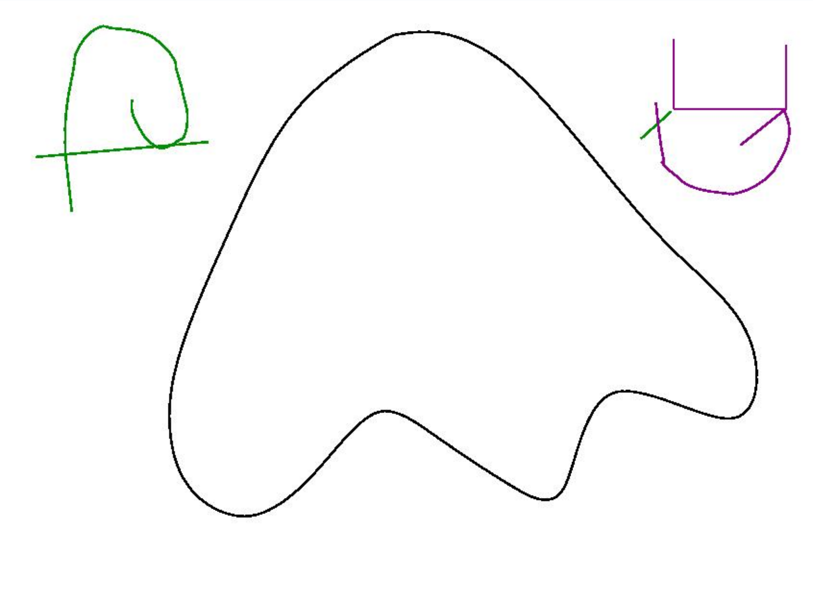

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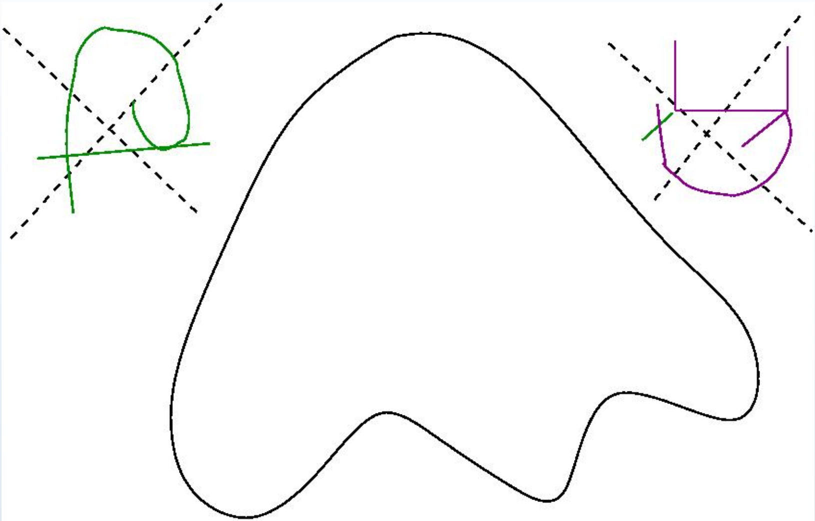

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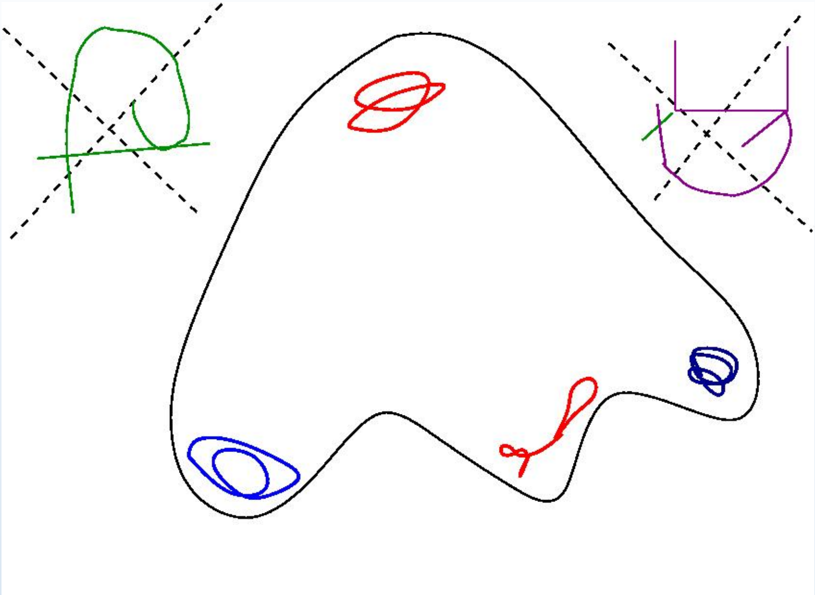

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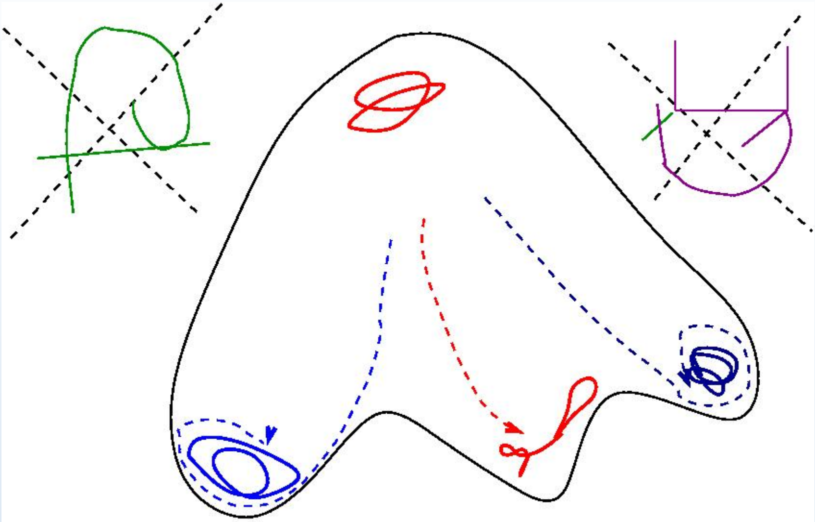
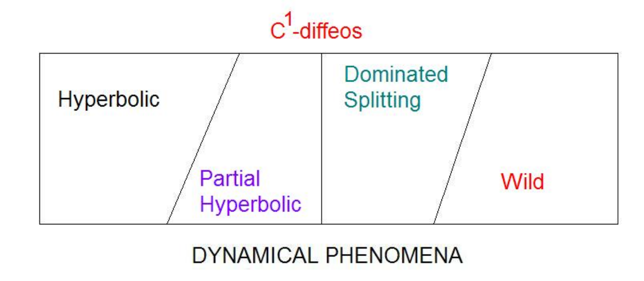


Other Dynamicals Phenomenas beyond Hyperbolicity



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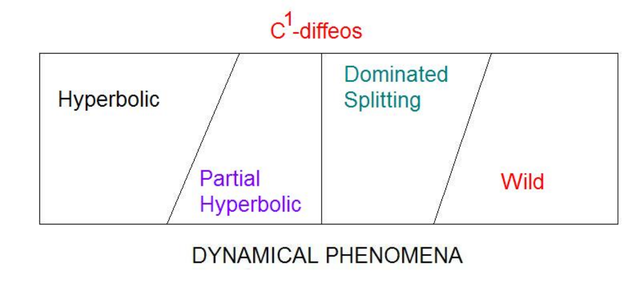

Which are the (semilocal) mechanisms involved in those phenomenas

Phenomenas and Mechanisms

Can we explain Dynamical Phenomenas through Homoclinic Mechanisms?.

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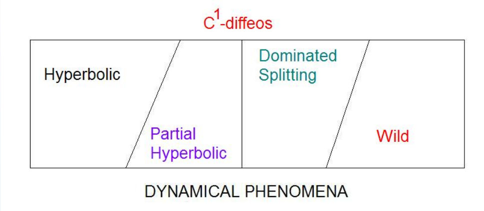
DICTIONARY BETWEEN MECHANISMS AND PHENOMENAS.

DICTIONARY: Phenomenas and Mechanisms



DICTIONARY: Phenomenas and Mechanisms